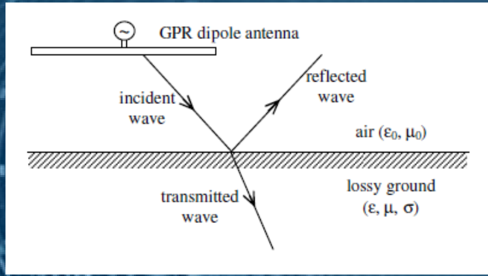
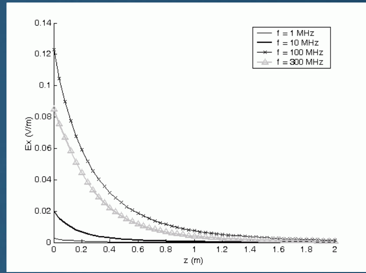




Department of Electronics
University of Split,
Split, Croatia

Frequency Domain Analysis of GPR Dipole Antenna using Galerkin-Bubnov Indirect Boundary Element Method

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

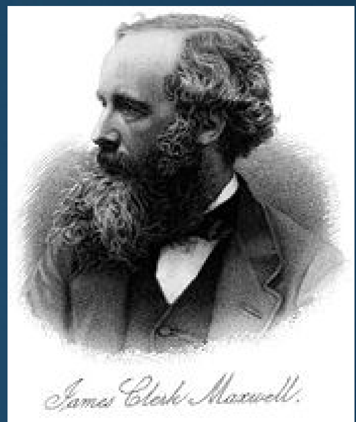
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

To be presented by

Dragan Poljak
University of Split, Croatia



Training School

Split, 11 November 2016

Department of Electronics
University of Split,
Split, Croatia

CONTENTS

- Introduction to Computational Electromagnetics (CEM) and Electromagnetic Compatibility (EMC)
- Frequency Domain Analysis of Wire Antennas
- Stochastic Modeling
- Computational Examples

Department of Electronics
University of Split,
Split, Croatia

Introduction to Computational Electromagnetics (CEM) and Electromagnetic Compatibility (EMC)

Historical note on modeling in electromagnetics

- Electromagnetics as a rigorous theory started when *James Clerk Maxwell* derived his celebrated four equations and published this work in the famous treatise in 1865.
- In addition to *Maxwell's equations* themselves, relating the behaviour of EM fields and sources we need:

  ✓ the constitutive relations of the medium

  ✓ the imposed boundary conditions of the physical problem of interest.

Department of Electronics
University of Split,
Split, Croatia

Introduction to Computational Electromagnetics (CEM) and Electromagnetic Compatibility (EMC)

Historical note on modeling in electromagnetics

- One of the first digital computer solution of the *Pocklington's equation* was reported in 1965.
- This was followed by the one of the first implementations of the **Finite Difference Method (FDM)** to the solution of partial differential equations in 1966 and time domain integral equation formulations in 1968 and 1973.
- Through 1970s the **Finite Element Method (FEM)** became widely used in almost all areas of applied EM applications.
- The **Boundary Element Method (BEM)** developed in the late seventies for the purposes of civil and mechanical engineering started to be used in electromagnetics in 1980s.

Department of Electronics
University of Split,
Split, Croatia

Introduction to Computational Electromagnetics (CEM) and Electromagnetic Compatibility (EMC)

EMC computational models and solution methods

- A basic EMC model, includes *EMI source* (any kind of undesired EMP), *coupling path* which is related to EM fields propagating in free space, material medium or conductors, and, finally, *EMI victim* - any kind of electrical equipment, medical electronic equipment (e.g. pacemaker), or even the human body itself.

EMI source ⟷ Coupling path ⟷ EMI victim

A basic EMC model

Department of Electronics
University of Split,
Split, Croatia

Introduction to Computational Electromagnetics (CEM) and Electromagnetic Compatibility (EMC)

EMC computational models and solution methods

- In principle, all EMC models arise from the rigorous EM theory concepts and foundations based on *Maxwell equations*.
- EMC models are analysed using either *analytical* or *numerical methods.*
- *Analytical models* are not useful for accurate simulation of electric systems, or their use is restricted to the solution of rather simplified geometries.
- More accurate simulation of various practical engineering problems is possible by the use of *numerical methods.*

Introduction to Computational Electromagnetics (CEM) and Electromagnetic Compatibility (EMC)

Classification of EMC models

- Regarding underlying *theoretical background* EMC models can be classified as:

  - ✓ circuit theory models featuring the concentrated electrical parameters
  - ✓ transmission line models using distributed parameters in which low frequency electromagnetic field coupling are taken into account
  - ✓ models based on the full-wave approach taking into account radiation effects for the treatment of electromagnetic wave propagation problems

Department of Electronics
University of Split,
Split, Croatia

Introduction to Computational Electromagnetics (CEM) and Electromagnetic Compatibility (EMC)

Summary remarks on EMC modeling

- The *main limits* to EMC modeling arise from the ***physical complexity*** of the considered electric system.

- Sometimes even the *electrical properties of the system* are too difficult to determine, or the number of independent parameters necessary for building a valid EMC model is too large for a practical computer code to handle.

Summary remarks on EMC modeling

- The advanced EMC modeling approach is based on ***integral equation formulations*** in the FD and TD and related *BEM* solution featuring the direct and indirect approach, respectively.
- This approach is preferred over a partial differential equation formulations and related numerical methods of solution, as the integral equation approach is based on the corresponding *fundamental solution of the linear operator* and, therefore, provides more accurate results.
- This *higher accuracy level* is paid with more complex formulation, than it is required within the framework of the partial differential equation approach, and related computational cost.

Department of Electronics
University of Split,
Split, Croatia

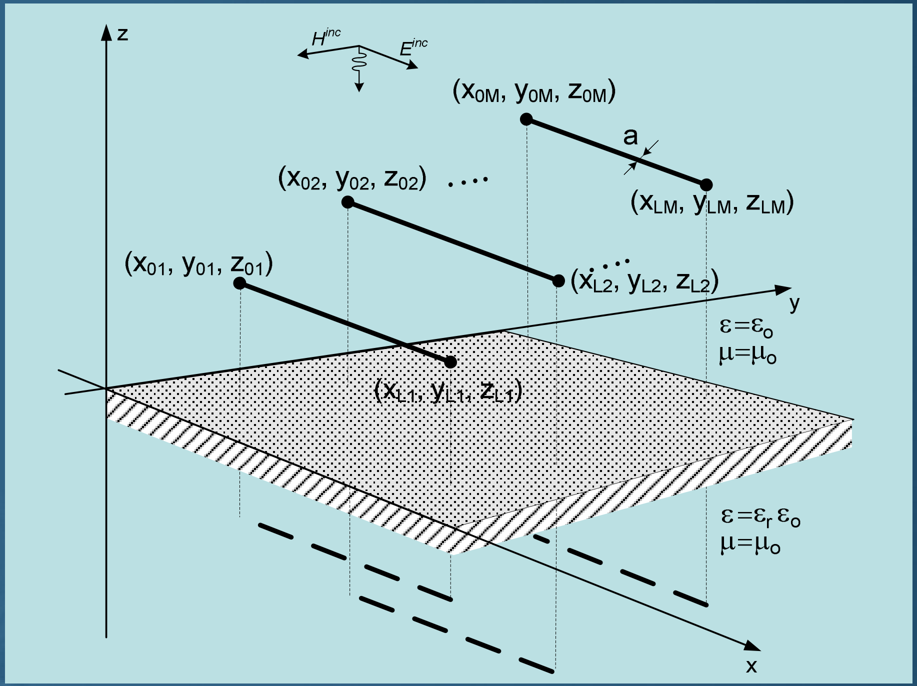
Frequency domain analysis of wire antennas

- In addition to antenna design the model of horizontal wires above lossy half-space has <u>numerous applications</u> in (EMC) in the analysis of aboveground lines and cables.
- The current distribution along the multiple wire structure is governed by the set of Pocklington equation for half-space problems.
- The influence of lossy half-space can be taken into account via the reflection coefficient (RC) approximation.

FD analysis of wire antennas

Department of Electronics
University of Split,
Split, Croatia

- The geometry of interest consists of *M* parallel straight wires horizontally placed above a lossy ground at height *h*.



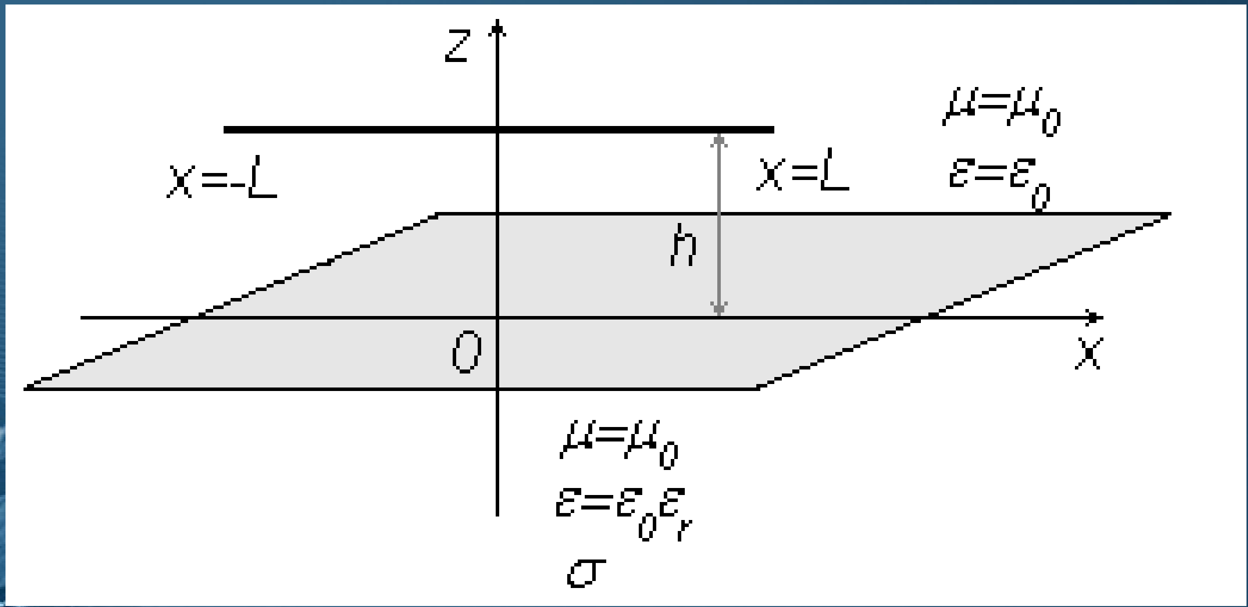
The geometry of the problem

- All wires are assumed to have same radius *a* and the length of the m-th wire is equal L_m.

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

The analysis starts by considering a single straight wire above a dissipative half-space.



Horizontal antenna over imperfect ground

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- The integral equation can be derived by enforcing the interface conditions for the *E*-field at the wire surface: $\vec{e}_x \cdot \left(\vec{E}^{exc} + \vec{E}^{sct}\right) = 0$

- The excitation represents the sum of the incident field and field reflected from the lossy ground: $\vec{E}^{exc} = \vec{E}^{inc} + \vec{E}^{ref}$

- The scattered field can be written as: $\vec{E}^{sct} = -j\omega\vec{A} - \nabla\varphi$

where **A** is the magnetic vector potential and ϕ is the scalar potential.

According to the thin wire approximation (TWA) only the axial component of the magnetic potential differs from zero:

$$E_x^{sct} = -j\omega A_x - \frac{\partial\varphi}{\partial x}$$

$$A_x = \frac{\mu}{4\pi}\int_0^L I(x')g(x,x')dx' \qquad \varphi(x) = \frac{1}{4\pi\varepsilon}\int_0^L q(x')g(x,x')dx'$$

while $q(x)$ is the charge distribution and $I(x')$ is the induced current along the wire.

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- Green function *g*(x,x') is given by:

$$g(x,x') = g_0(x,x') - R_{TM}\, g_i(x,x')$$

where $g_0(x, x')$ is the free space-Green function and $g_i(x, x')$ arises from the image theory:

$$g_o(x,x') = \frac{e^{-jk_o R_o}}{R_o} \qquad g_i(x,x') = \frac{e^{-jk_o R_i}}{R_i}$$

R_o and R_i , respectively, is the distance from the source to the observation point, and the reflection coefficient is

$$R_{TM} = \frac{n\cos\Theta - \sqrt{n - \sin^2\Theta}}{n\cos\Theta + \sqrt{n - \sin^2\Theta}} \qquad n = \varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0} \qquad \Theta = \operatorname{arctg}\frac{|x - x'|}{2h}$$

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- The linear charge density and the current distribution along the line are related through the equation of continuity:

$$q = -\frac{1}{j\omega}\frac{dI}{dx}$$

- After mathematical manipulation it follows:

$$\varphi(x) = -\frac{1}{j4\pi\omega\varepsilon}\int_0^L \frac{\partial I(x')}{\partial x'} g(x,x')dx'$$

leading to the following integral relationship for the scattered field:

$$E_x^{sct} = -j\omega\frac{\mu}{4\pi}\int_0^L I(x')g(x,x')dx' + \frac{1}{j4\pi\omega\varepsilon}\frac{\partial}{\partial x}\int_0^L \frac{\partial I(x')}{\partial x'} g(x,x')dx'$$

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- Combining previous equations results in the following integral equation for the current distribution induced along the wire:

$$E_x^{exc} = j\omega \frac{\mu}{4\pi} \int_0^L I(x') g(x,x') dx' - \frac{1}{j4\pi\omega\varepsilon} \frac{\partial}{\partial x} \int_0^L \frac{\partial I(x')}{\partial x'} g(x,x') dx'$$

- This equation is well-known in antenna theory representing one of the most commonly used variants of the Pocklington's integro-differential equation for half space problems.

- This integro-differential equation is particularly attractive for numerical modeling, as there is no second-order differential operator under the integral sign.

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- The electric field components are:

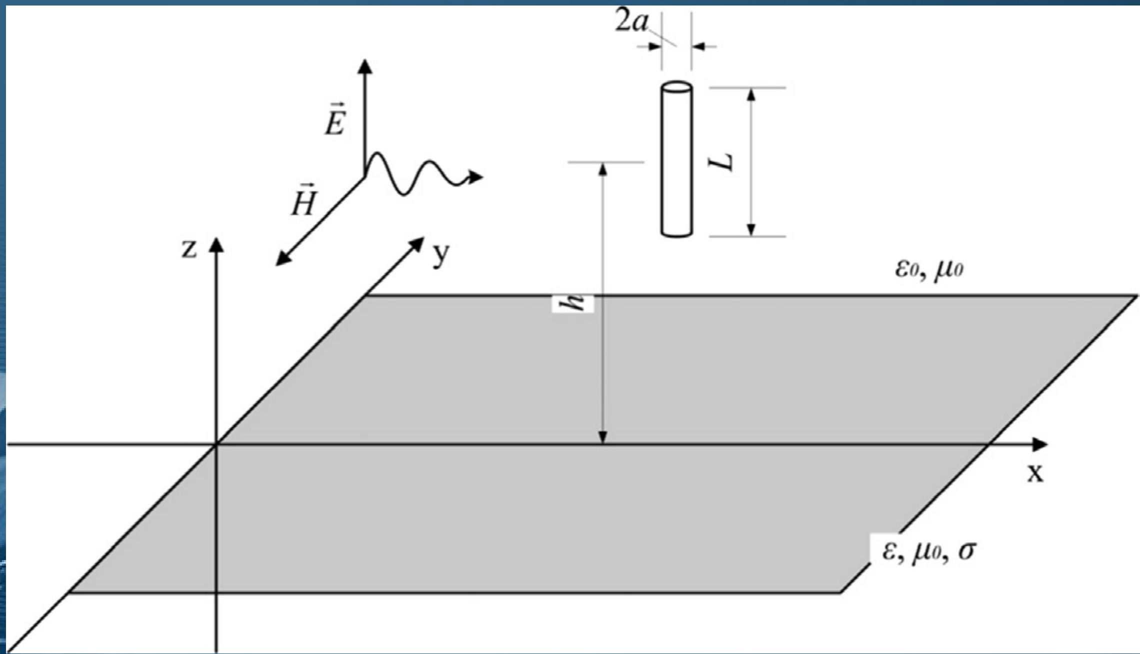
$$E_x = \frac{1}{j4\pi\omega\varepsilon_0}\left[-\int_{-L}^{L}\frac{\partial I(x')}{\partial x'}\frac{\partial g(x,x')}{\partial x'}dx' + k^2\int_{-L}^{L} g(x,x')I(x')dx'\right]$$

$$E_z = \frac{1}{j4\pi\omega\varepsilon_0}\int_{-L}^{L}\frac{\partial I(x')}{\partial x'}\frac{\partial g(x',z)}{\partial z}dx'$$

$$E_y = \frac{1}{j4\pi\omega\varepsilon_0}\int_{-L}^{L}\frac{\partial I(x')}{\partial x'}\frac{\partial g(x',y)}{\partial y}dx'$$

FD analysis of wire antennas

- Vertical wire above a real ground



FD analysis of wire antennas

- Integro-differential equation for vertical wire

$$\left[\frac{\partial^2}{\partial z^2}+k^2\right]\int_{h-(L/2)}^{h+(L/2)} I(z')g(z,z')dz' = -j4\pi\frac{k}{Z_0}E_z^{exc}$$

The propagation constant k is given by

$$k=\omega\sqrt{\mu_0\varepsilon_0}$$

and Z_0 is the free space impedance

$$Z_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

The total Green function is, as follows

$$g(z,z')=g_0(z,z')-\Gamma_{Fr}^{ref}g_i(z,z')$$

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- Vertical wire penetrating the ground

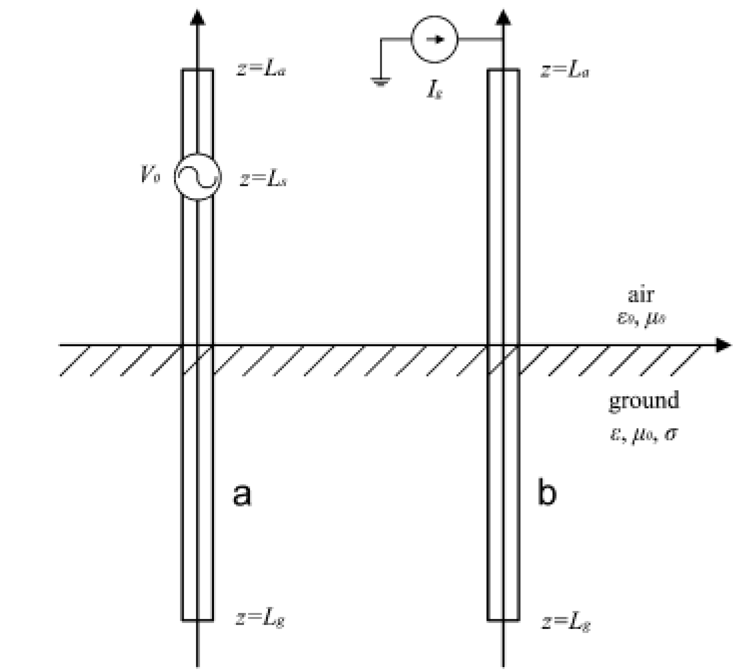


Fig. 4. Vertical antenna penetrating the ground excited by a voltage source (a) and by a current source (b).

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- Integro-differential equation for vertical penetrating the ground

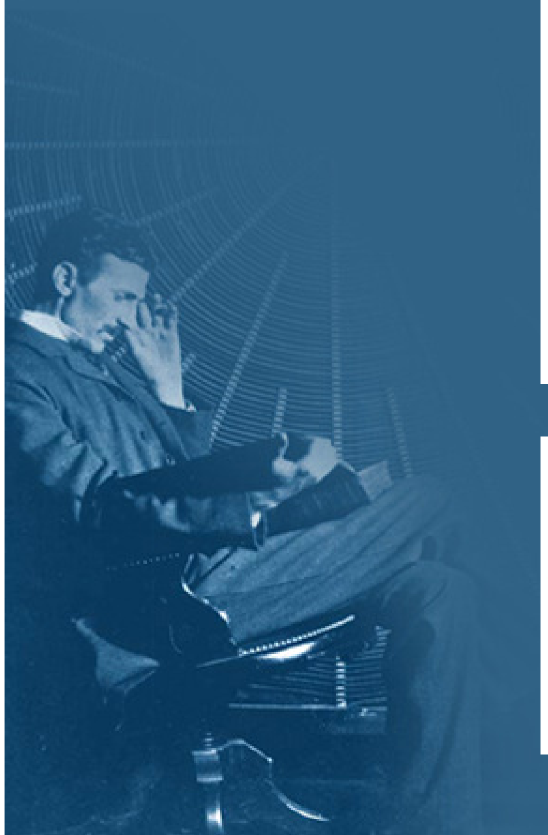
$$E_z^{inc} = -\frac{1}{j4\pi\omega\varepsilon_{eff}}\begin{bmatrix} \int_{-L_g}^{0}\left(\frac{\partial^2}{\partial z^2}+k_2^2\right)G^{22}(\rho,z,z')I(z')dz' + \\ \int_{0}^{L_a}\left(\frac{\partial^2}{\partial z^2}+k_2^2\right)G^{12}(\rho,z,z')I(z')dz' \end{bmatrix}; \quad z \le 0,$$

$$E_z^{inc} = -\frac{1}{j4\pi\omega\varepsilon_0}\begin{bmatrix} \int_{-L_g}^{0}\left(\frac{\partial^2}{\partial z^2}+k_1^2\right)G^{21}(\rho,z,z')I(z')dz' + \\ \int_{0}^{L_a}\left(\frac{\partial^2}{\partial z^2}+k_1^2\right)G^{11}(\rho,z,z')I(z')dz' \end{bmatrix}; \quad z > 0,$$

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- Integro-differential equation for vertical penetrating the ground



where

$$G^{11}(\rho,z,z') = g_0(z,z') - g_i(z,z') + g_{Sa}(\rho,z,z'), \text{ for points } z>0 \text{ and } z'>0$$

while g_0, g_i are defined by (13) and g_{Sa} is given by

$$g_{Sa}(\rho,z,z') = 2\int_0^{\infty} J_0(\lambda\rho)e^{-\mu(z+z')}\frac{\varepsilon_{eff}}{\varepsilon_{eff}\mu+\varepsilon_0\mu_E}\lambda d\lambda$$

with

$$\mu=\sqrt{\lambda^2-k^2};\ \mu_E=\sqrt{\lambda^2-k_2^2};\ k_2^2=\underline{n}k^2 \qquad (21)$$

where $\underline{n}$ is the relative complex permittivity of the air–ground interface given by (14), while ε_{eff} is the complex permittivity of the ground determined by (15), and k is wave propagation of free space

FD analysis of wire antennas

Department of Electronics
University of Split,
Split, Croatia

- Integro-differential equation for vertical penetrating the ground

Furthermore

$$G^{22}(\rho,z,z') = g_0(z,z') - g_i(z,z') + g_{Su}(\rho,z,z'), \text{ for points } z<0 \text{ and } z'<0 \tag{22}$$

while g_0, g_i, g_{Su} are defined, as follows:

$$g_0(z,z') = \frac{e^{-jk_2R}}{R}, \quad g_i(z,z') = \frac{e^{-jk_2R_i}}{R_i} \tag{23}$$

$$g_{Su}(\rho,z,z') = 2\int_0^\infty J_0(\lambda\rho)e^{-\mu_E(z+z')}\frac{\varepsilon_0}{\varepsilon_0\mu_E+\varepsilon_{eff}\mu}\lambda d\lambda \tag{24}$$

The Green functions related to transmitted field are given by:

$$G^{12}(\rho,z,z') = 2\int_0^\infty J_0(\lambda\rho)e^{-\mu_E|z|}e^{-\mu|z'|}\frac{\varepsilon_{eff}}{\varepsilon_{eff}\mu+\varepsilon_0\mu_E}\lambda d\lambda \tag{25}$$

for points $z<0$ and $z'>0$, and

$$G^{21}(\rho,z,z') = 2\int_0^\infty J_0(\lambda\rho)e^{-\mu_E|z|}e^{-\mu|z'|}\frac{\varepsilon_0}{\varepsilon_{eff}\mu+\varepsilon_0\mu_E}\lambda d\lambda \tag{26}$$

for points $z>0$ and $z'<0$:

Sommerfeld integrals (20), (24)–(28) are evaluated numerically using Simpson adaptive quadrature in complex plane [21].

Furthermore, certain continuity conditions have to be satisfied at the air–ground interface, i.e.:

$$I(z=0^+) = I(z=0^-) \tag{27}$$

$$\frac{\partial I(z=0^+)}{\partial z}\varepsilon_{eff} = \frac{\partial I(z=0^-)}{\partial z}\varepsilon_0 \tag{28}$$

where $(+)$ and $(-)$ denote above and below the interface, respectively.

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- An extension to the wire array is straightforward and results in the the set of coupled Pocklington integral equations:

$$E_x^{exc} = -\frac{1}{j4\pi\omega\varepsilon_0}\sum_{n=1}^{M}\int_{-L_n/2}^{L_n/2}\left[\frac{\partial^2}{\partial x^2}+k_1^2\right]\left[g_{0mn}(x,x') - R'_{TM}\, g_{imn}(x,x')\right] I_n(x')dx'$$

$$m = 1, 2, \ldots M$$

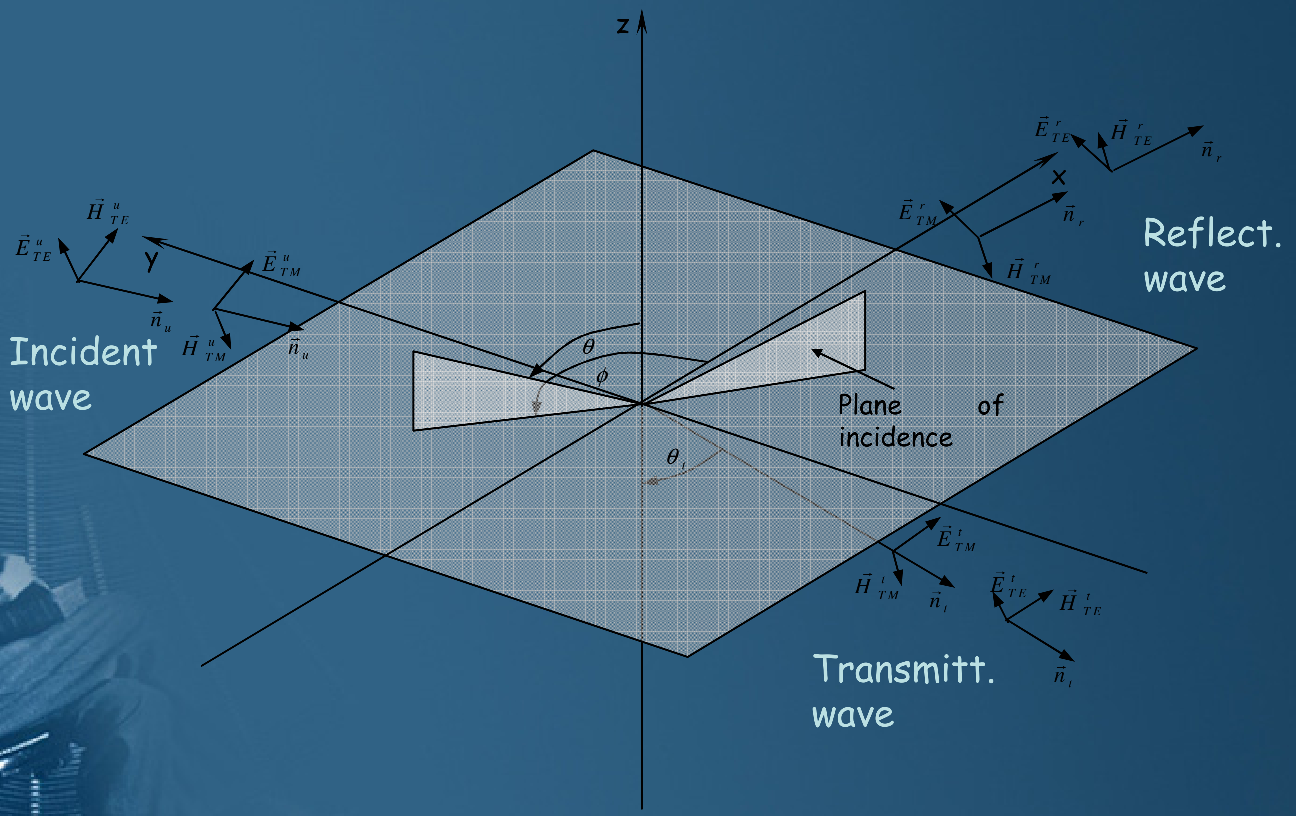
where $I_n(x')$ is the unknown current distribution induced on the n-th wire axis, $g_{0mn}(x,x`)$ is the free space Green function, while $g_{imn}(x,x`)$ *arises from* the image theory:

$$g_{0mn}(x,x') = \frac{e^{-jk_1 R_{1mn}}}{R_{1mn}}$$

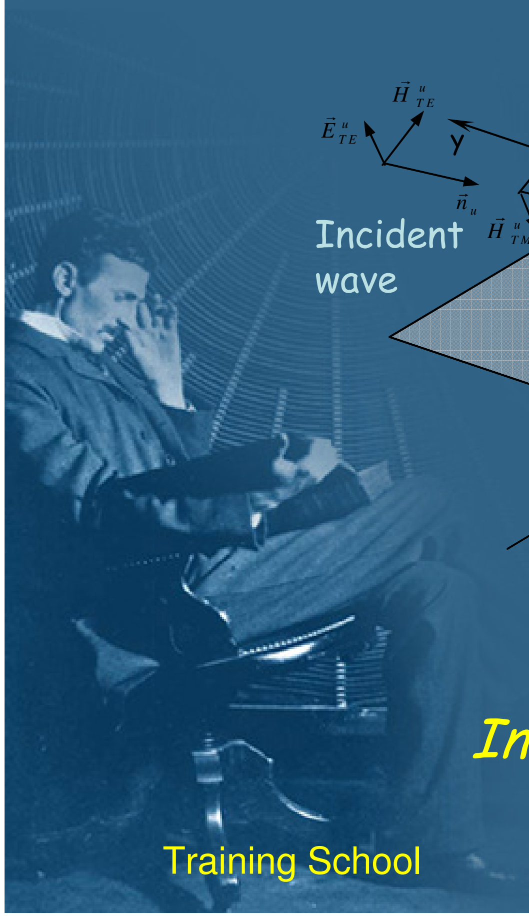
$$g_{0mn}(x,x') = \frac{e^{-jk_1 R_{1mn}}}{R_{1mn}}$$

FD analysis of wire antennas

- The wires are excited by a plane wave of arbitrary incidence



Incident, reflected and transmitted wave



Department of Electronics
University of Split,
Split, Croatia

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- The tangential component of an incident plane wave can be represented in terms of its vertical E_V and horizontal E_H component:

$$
\begin{aligned}
E_x^{exc} &= E_x^i + E_x^r = \\
&E_0(\sin\alpha\sin\phi - \cos\alpha\cos\theta\cos\phi)e^{-jk_1\vec{n}_i\cdot\vec{r}} + \\
&+E_0\left(R_{TE}\sin\alpha\sin\phi + R_{TM}\cos\alpha\cos\theta\cos\phi\right)e^{-jk_1\vec{n}_r\cdot\vec{r}}
\end{aligned}
$$

where α is an angle between *E-field* vector and the plane of incidence.

R_{TM} and R_{TE} are the vertical and horizontal Fresnel reflection coefficients at the air-earth interface given by:

$$R_{TM} = \frac{\underline{n}\cos\theta - \sqrt{\underline{n} - \sin^2\theta}}{\underline{n}\cos\theta + \sqrt{\underline{n} - \sin^2\theta}}$$

$$R_{TE} = \frac{\cos\theta - \sqrt{\underline{n} - \sin^2\theta}}{\cos\theta + \sqrt{\underline{n} - \sin^2\theta}}$$

$$
\begin{aligned}
\vec{n}_i\cdot\vec{r} &= -x\sin\theta\cos\phi - y\sin\theta\sin\phi - z\cos\theta \\
\vec{n}_r\cdot\vec{r} &= -x\sin\theta\cos\phi - y\sin\theta\sin\phi + z\cos\theta
\end{aligned}
$$

FD analysis of wire antennas

- The *E*-field components are given, as follows:

$$E_x = \frac{1}{j4\pi\omega\varepsilon_0}\sum_{n=1}^{M}\left[-\int_{-L_n}^{L_n}\frac{\partial I_n(x')}{\partial x'}\frac{\partial G_{nm}(x,x')}{\partial x'}dx' + k^2\int_{-L_n}^{L_n}G_{nm}(x,x')I_n(x')dx'\right]$$

$$E_y = \frac{1}{j4\pi\omega\varepsilon_0}\sum_{n=1}^{M}\int_{-L_n}^{L_n}\frac{\partial I_n(x')}{\partial x'}\frac{\partial G_{nm}(x,x')}{\partial y}dx'$$

$$E_z = \frac{1}{j4\pi\omega\varepsilon_0}\sum_{n=1}^{M}\int_{-L_n}^{L_n}\frac{\partial I_n(x')}{\partial x'}\frac{\partial G_{nm}(x,x')}{\partial z}dx'$$

where *m=1, 2, …, M* and Green function *G* is given by:

$$G_{nm}(x,x') = g_{0nm}(x,x') - R_{TM}\, g_{inm}(x,x')$$

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

BEM solution of Pocklington equation system

- The BEM procedure starts, as follows:

$$I(x') = I_{1i}\frac{x_{2i} - x'}{\Delta x} + I_{2i}\frac{x' - x_{1i}}{\Delta x}$$

- Performing certain mathematical manipulations and BEM discretisation results in the following matrix equation:

$$\sum_{k=1}^{N_e}[Z]_{pk}\{I\}_k = \{V\}_p$$

N_e - the total number of elements

p=1,2,…,M

$[Z]_{pk}$ - the interaction matrix:

$$[Z]_{pk}^{e} = -\int_{\Delta l_p}\int_{\Delta l_k}\{D\}_p\{D'\}_k^T g_{ji}(x,x')dx'dx + k^2\int_{\Delta l_p}\int_{\Delta l_k}\{f\}_l\{f'\}_k^T g_{ji}(x,x')dx'dx$$

- Vectors $\{f\}$and $\{f'\}$ contain shape functions $f_n(x)$ and $f_n(x')$, while $\{D\}$ and $\{D'\}$ contain their derivatives.

- The vector $\{V\}_p$ represents the voltage along the segment:

$$\{V\}_p = -j4\pi\omega\varepsilon_0\int_{\Delta l_p}E_x^{inc}(x)\{f\}_p\,dx$$

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

The BEM field calculation

- Applying the BEM formalism to field expressions it follows:

$$E_x = \frac{1}{j4\pi\omega\varepsilon_0}\sum_{n=1}^{M}\sum_{i=1}^{N_j}\left[-\frac{I_{i+1,n}-I_{i,n}}{\Delta x}\int_{x_{i,n}}^{x_{i+1,n}}\frac{\partial G_{nm}(x,x')}{\partial x'}dx' + k^2\int_{x_{i,n}}^{x_{i+1,n}} G_{nm}(x,x')I_{in}(x')dx'\right];$$
$$m = 1,2,\ldots,M$$

$$E_y = \frac{1}{j4\pi\omega\varepsilon_0}\sum_{n=1}^{M}\sum_{i=1}^{N_j}\frac{I_{i+1,n}-I_{i,n}}{\Delta x}\int_{x_{i,n}}^{x_{i+1,n}}\frac{\partial G_{nm}(x,x')}{\partial y}dx'; \quad m = 1,2,\ldots,M$$

$$E_z = \frac{1}{j4\pi\omega\varepsilon_0}\sum_{n=1}^{M}\sum_{i=1}^{N_j}\frac{I_{i+1,n}-I_{i,n}}{\Delta x}\int_{x_{i,n}}^{x_{i+1,n}}\frac{\partial G_{nm}(x,x')}{\partial z}dx'; \quad m = 1,2,\ldots,M$$

- N_j is the total number of boundary elements on the *j*-th wire

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

Computational examples

Vertical wire:

- Single wire above a lossy ground
- Wire penetrating the interfacearray above a lossy ground

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

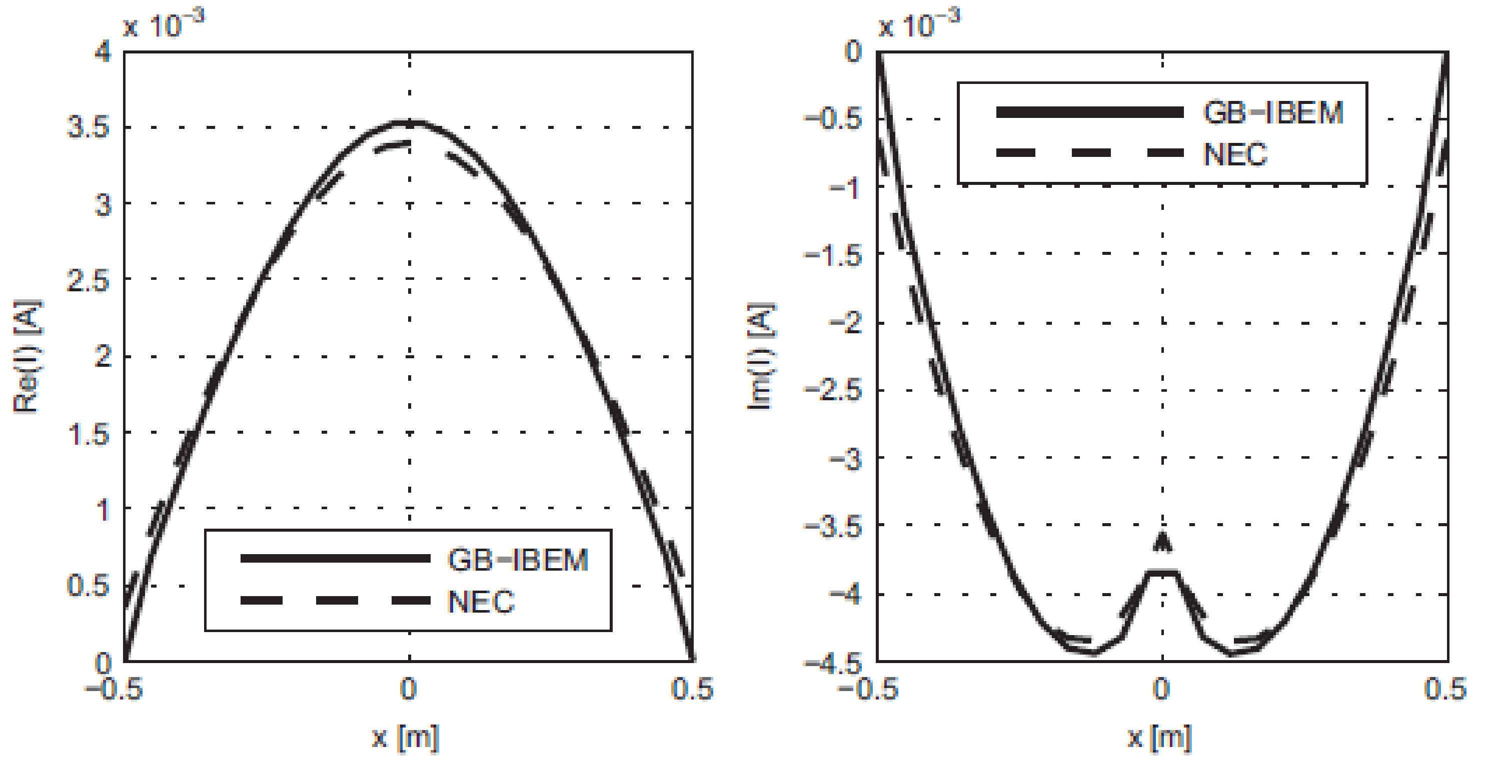


Fig. 2. Current distribution ($L=1$ m, $a=0.005$ m, $h=2$ m, $\sigma=1$ mS/m, $\varepsilon_r=10$, $V_0=1$ V, $f=168.2$ MHz).

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

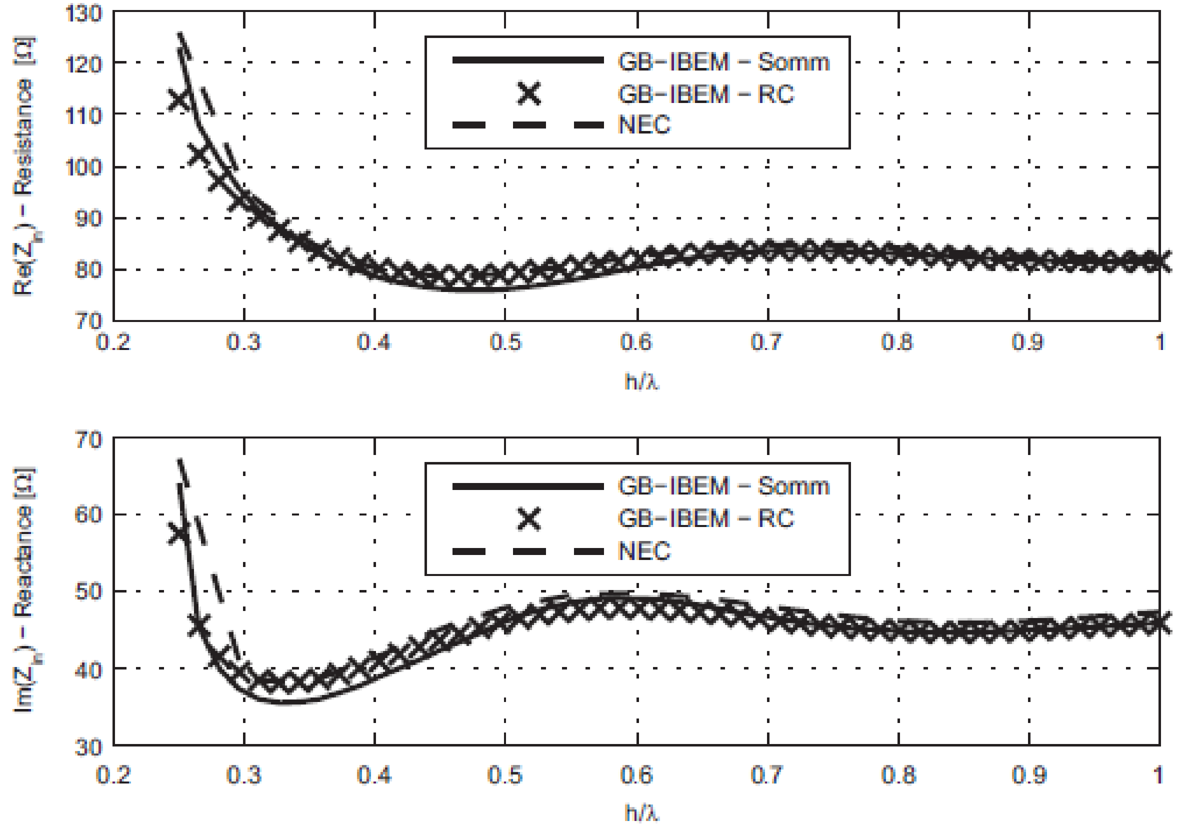


Fig. 3. Impedance (real and imaginary part) of vertical dipole above a real ground ($L=1$ m, $a/\lambda=0.0005$, $\sigma=3$ mS/m, $\varepsilon_r=10$, $f=3$ MHz).

FD analysis of wire antennas

24 D. Poljak et al. / Engineering Analysis with Boundary Elements 50 (2015) 19–28

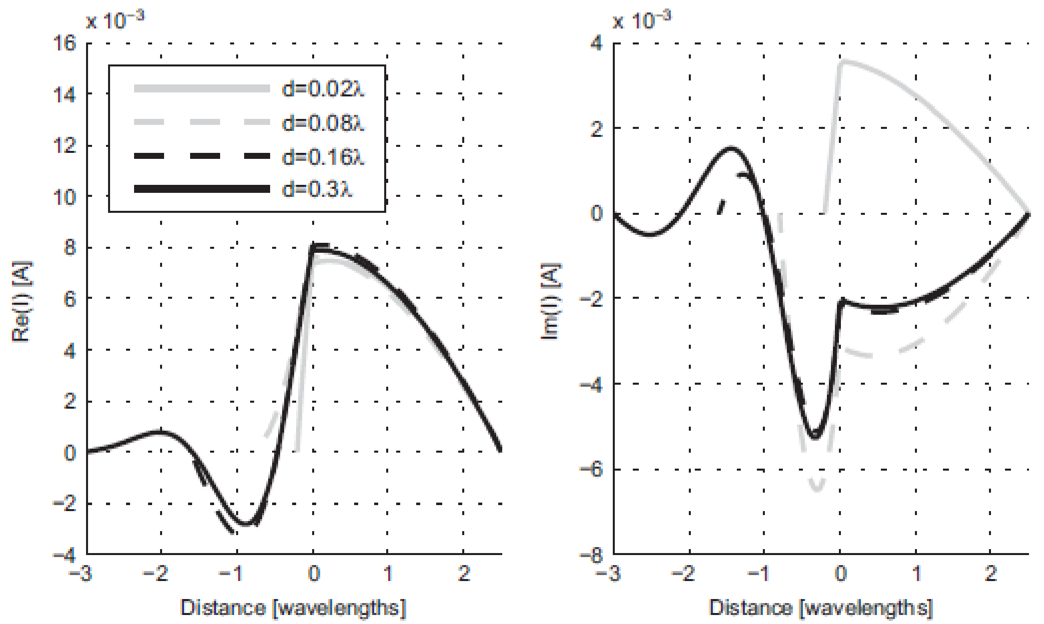


Fig. 5. Current distribution induced along the vertical wire penetrating a ground for various lengths of ground stake and voltage source at bottom end of the air stake.

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

Computational examples

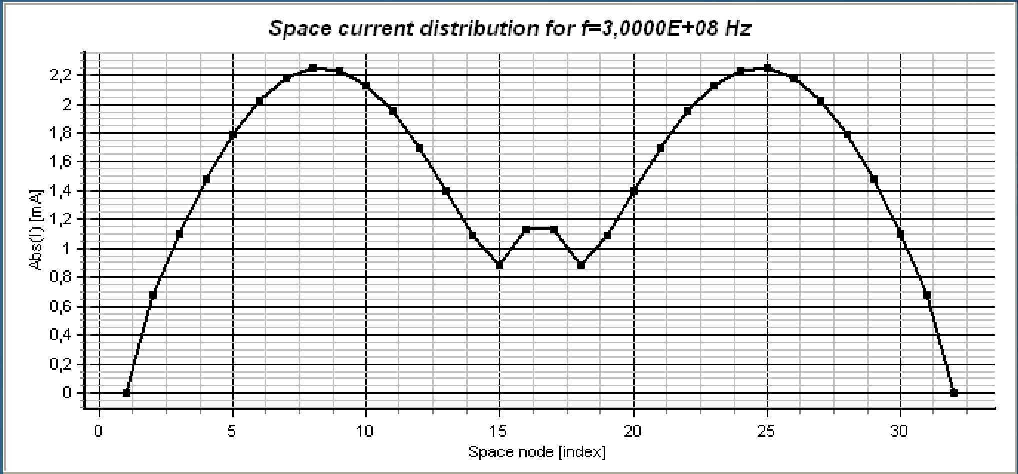
Numerical results are obtained via TWiNS code for:

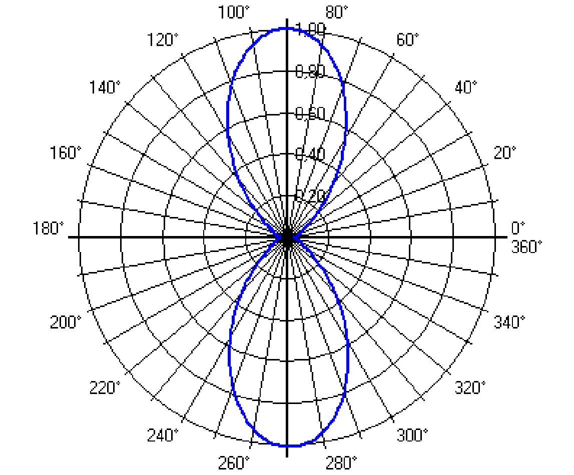
- Single wire above a lossy ground
- Wire array above a lossy ground
- **Practical example: Yagi-Uda array for VHF TV applications**
- **Practical example: single LPDA for ILS**

Department of Electronics
University of Split,
Split, Croatia

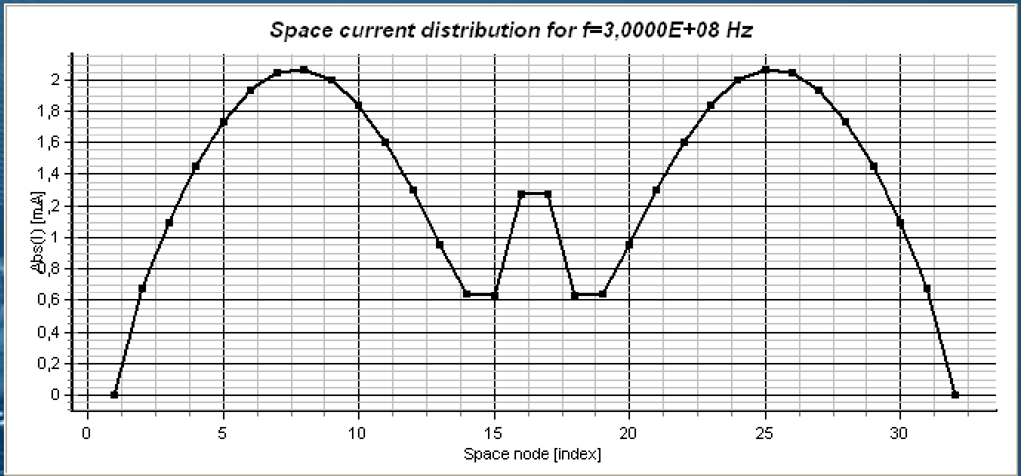
FD analysis of wire antennas

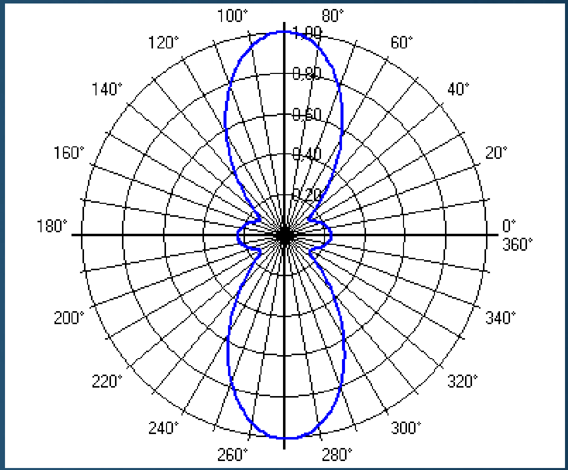
h=1m





h=0.2m



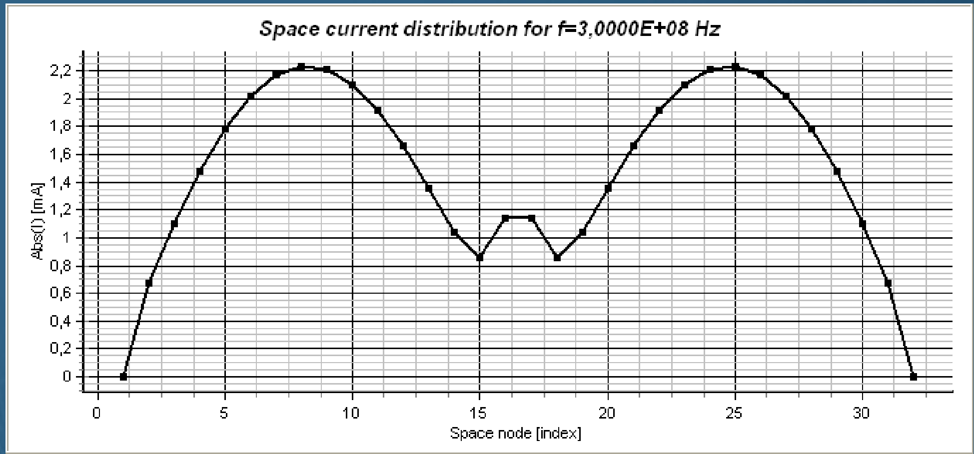


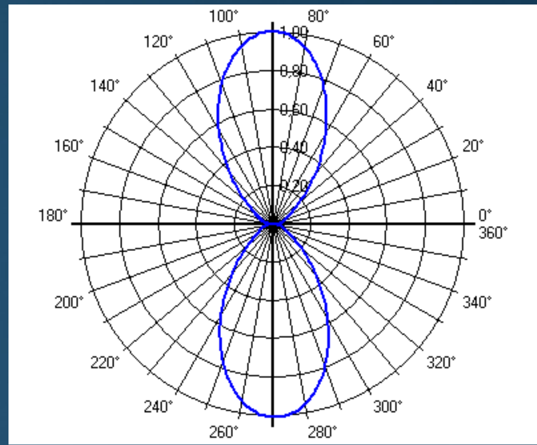
Dipole above a PEC ground, f=300MHz, L=λ

Department of Electronics
University of Split,
Split, Croatia

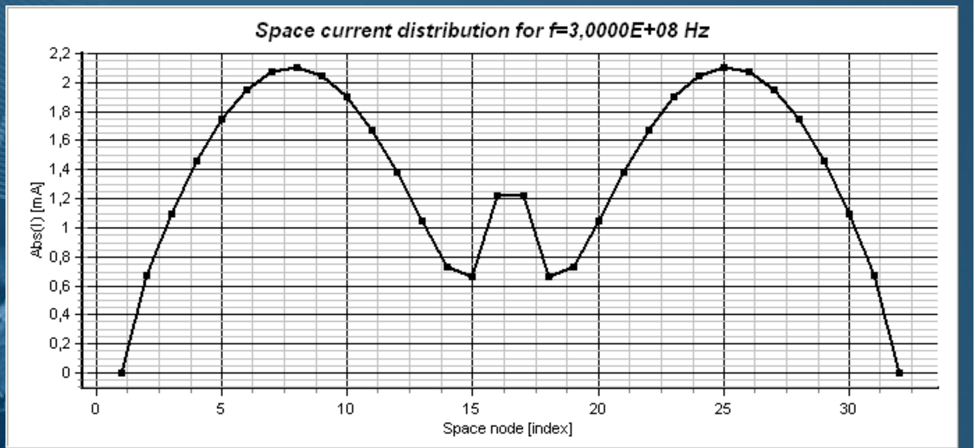
FD analysis of wire antennas

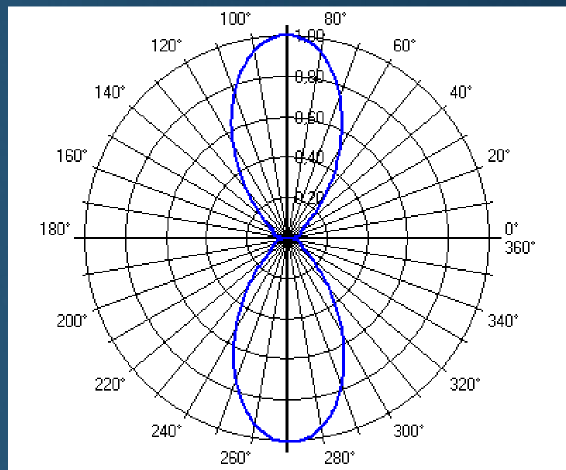
h=1m





h=0.2m



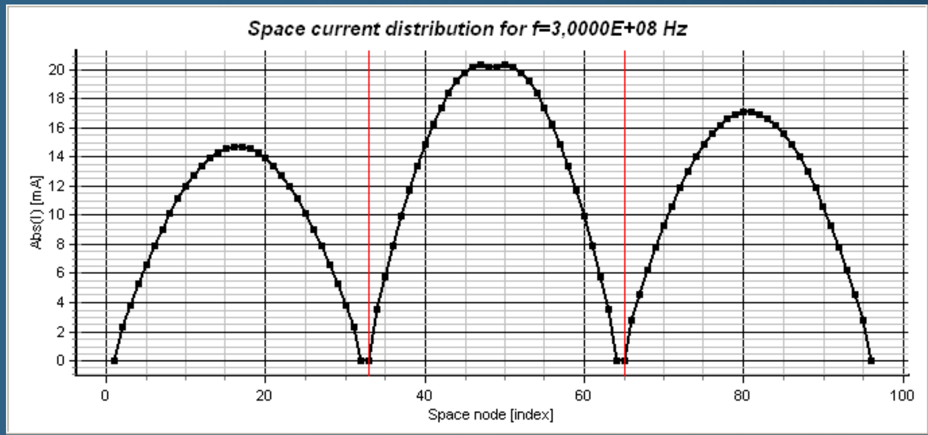


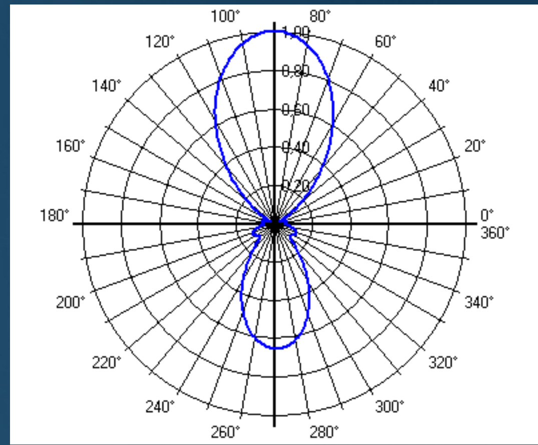
Dipole above a <u>lossy ground</u>, f=300MHz, L=λ/2, ε_r= 30, σ=0.04 S/m

Department of Electronics
University of Split,
Split, Croatia

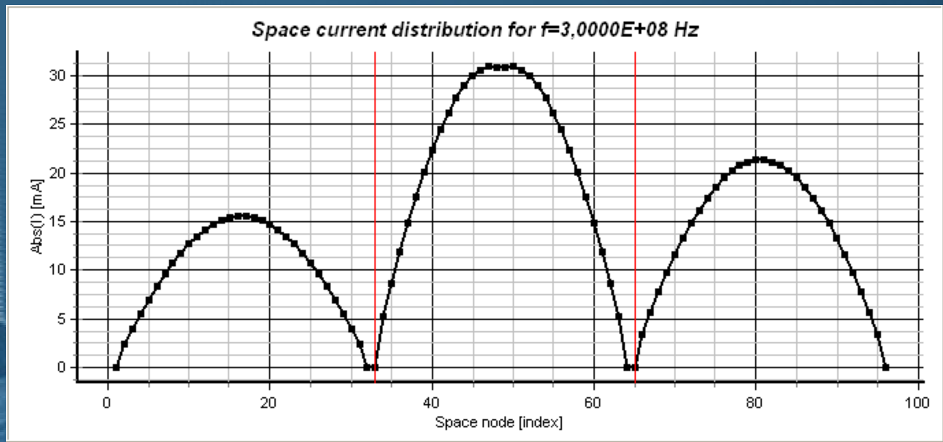
FD analysis of wire antennas

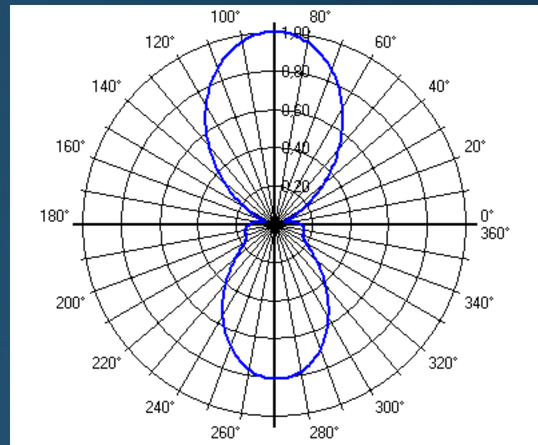
h=1m





h=0.2m



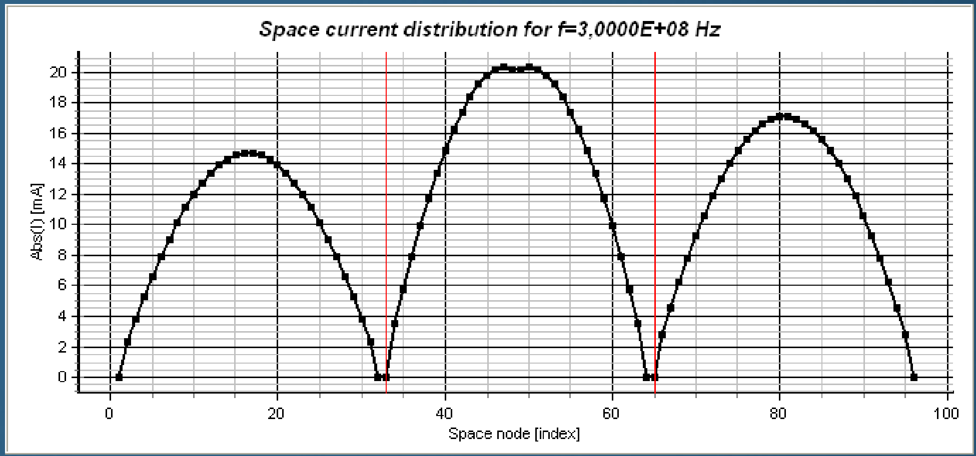


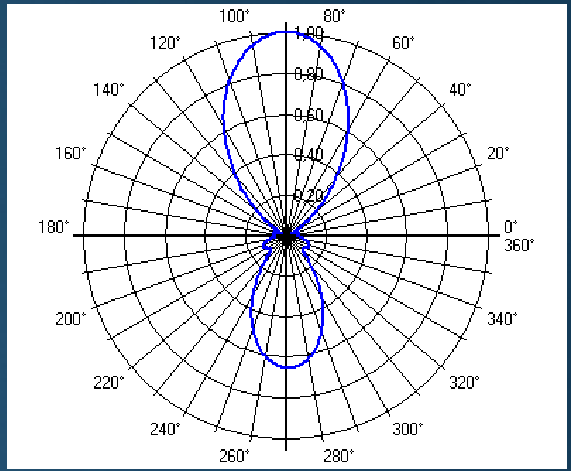
XYplane: Currents and far-field pattern for the Yagi-Uda array above a PEC ground (reflector, fed element + director), $a=0.0025m$, $L_r=0.479m$, $L_f=0.453m$ i $L_d=0.451m$, $d=0.25m$ $V_g=1V$

FD analysis of wire antennas

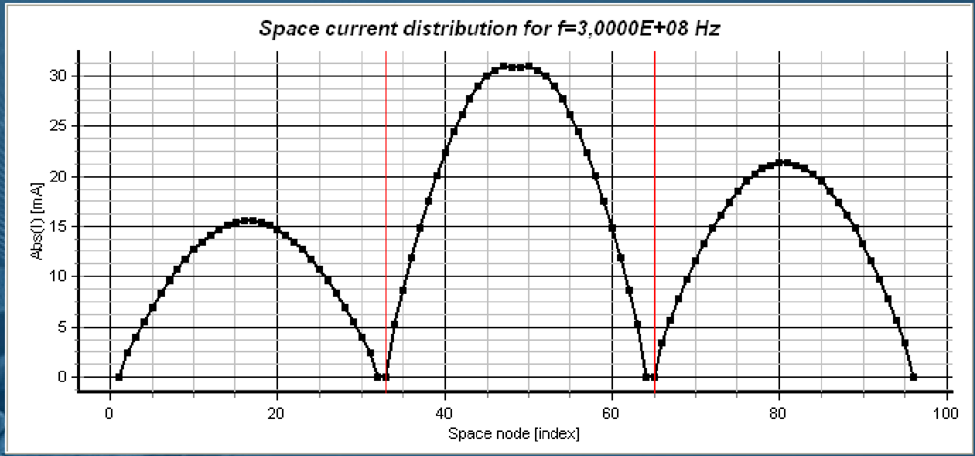
Department of Electronics
University of Split,
Split, Croatia

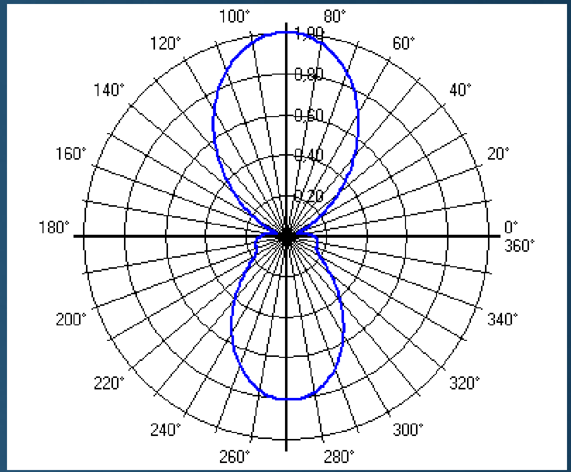
h=1m





h=0.2m



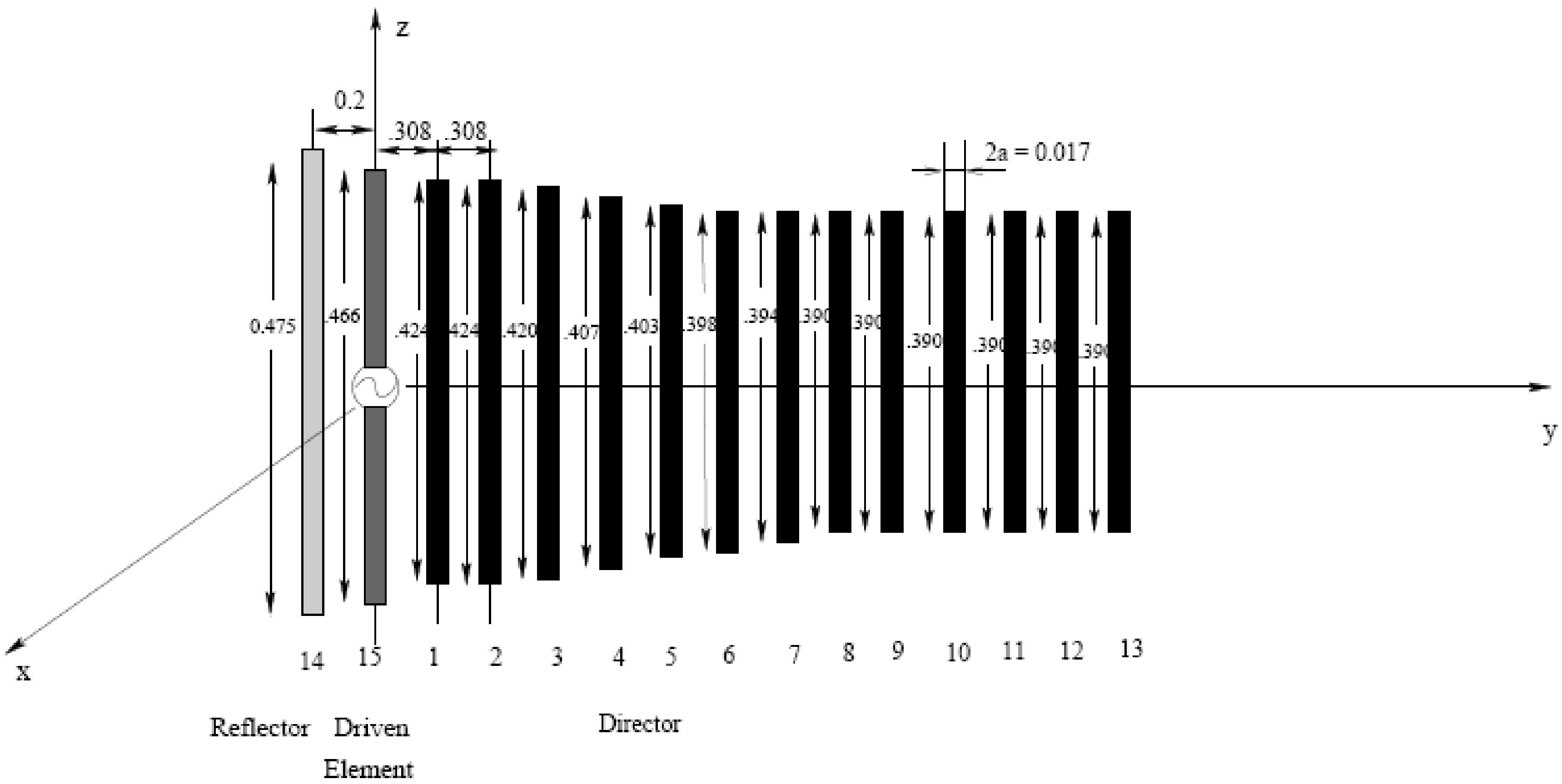


XYplane: Currents and far-field pattern for the Yagi-Uda array above a real ground (reflector, fed element + director),
$a=0.0025m$, $L_r=0.479m$, $L_f=0.453m$ i $L_d=0.451m$, $d=0.25m$ $V_g=1V$

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

Yagi-Uda array for VHF TV applications



Geometry of Yagi-Uda array with 15 elements

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

Yagi-Uda array: technical parameters

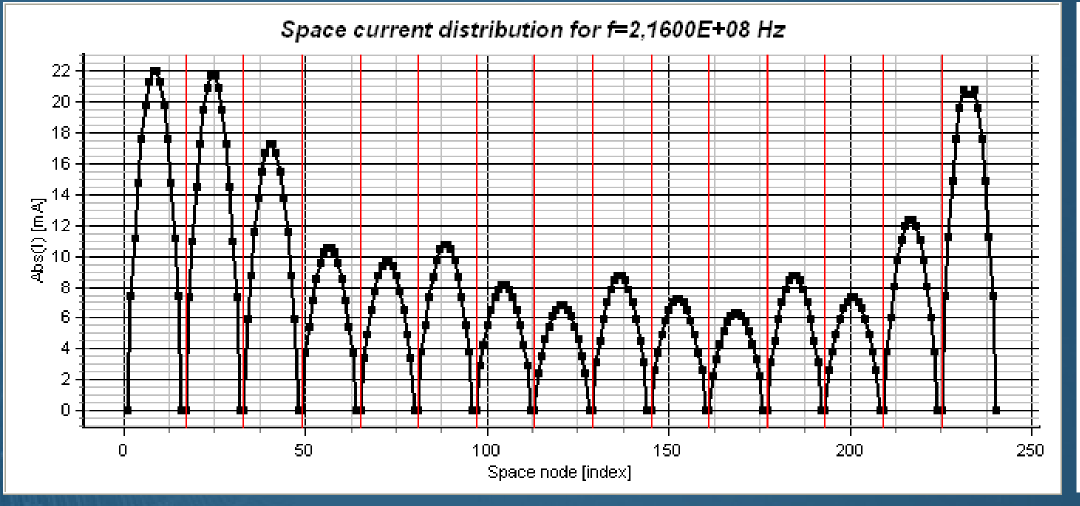
- Number of wires N=15
- Number of directors 13
- Operating frequency f=216MHz (frequency of 13th TV channel)
- Wire radius: a= 0.0085λ=0.0118m
- *Director lengths* $l_1=l_2=0.424\lambda=0.589$m, $l_3=0.420\lambda=0.583$m,
- $l_4=0.407\lambda=0.565$m, $l_5=0.403\lambda=0.56$m, $l_6=0.398\lambda=0.553$m,
- $l_7=0.394\lambda=0.547$m, l_8- $l_{13}=0.390\lambda=0.542$m
- *Reflector lengths* $l_{14}=0.475\lambda=0.66$m
- *fed-element length* $l_{15}=0.466\lambda=0.647$m
- *Distance between directors* $d_d=0.308\lambda=0.427$m
- *Distance between reflector and fed-element* $d_r=0.2\lambda=0.278$m

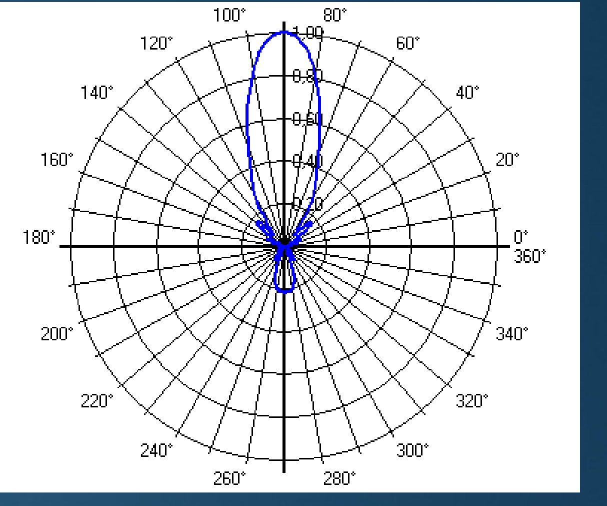
Computational aspects

- $\Delta l \geq 2a$
- L_{tot}= 5.83m, N_{tot}=225

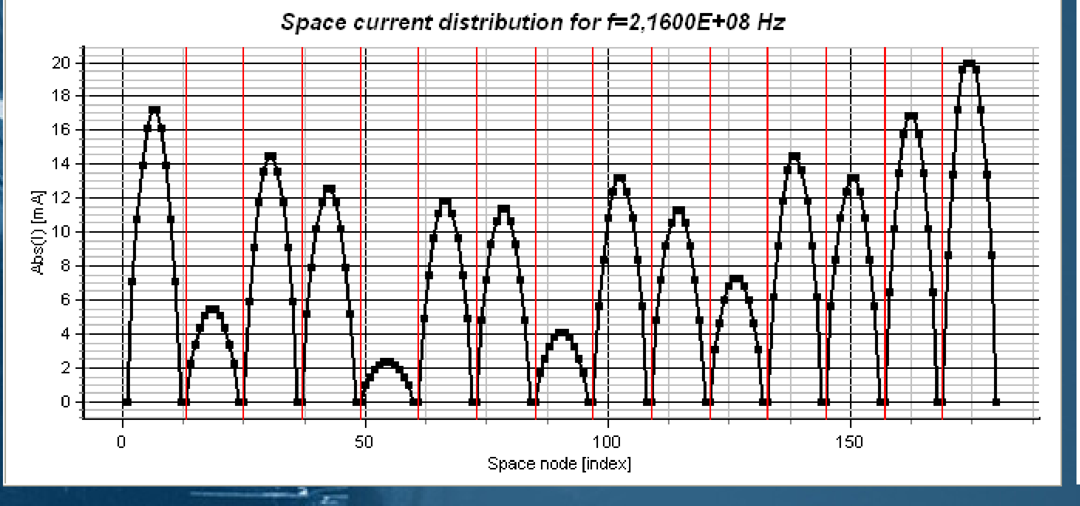
Department of Electronics
University of Split,
Split, Croatia

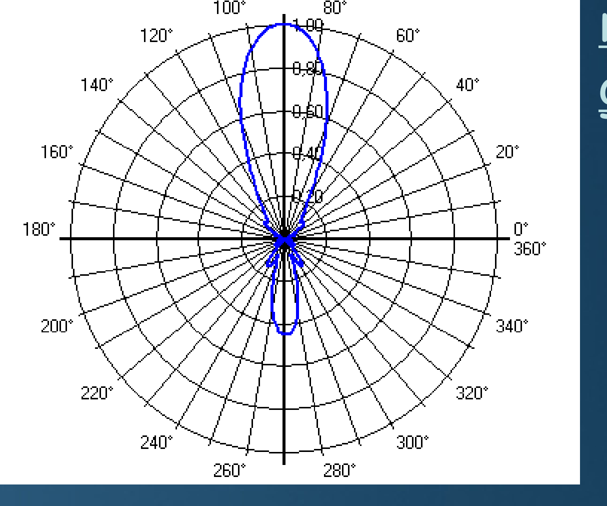
FD analysis of wire antennas





free space





real ground

XYplane: Currents and far-field pattern for the Yagi-Uda array

Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

Log-periodic dipole array

- LPDA impedance and radiation properties repeat periodically as the logarithm of frequency (VHF and UHF bands; 30MHz to 3GHz).
- The LPDA antennas are easy to optimize, while the crossing of the feeder between each dipole element leads to a mutual cancellation of backlobe components from the individual elements yielding to a very low level of backlobe radiation (around 25dB below main lobe gain at HF and 35dB at VHF and UHF).

The cutoff frequencies of the truncated structure is determined by the electrical lengths of the largest and shortest elements of the structure.

- The use of logarithmic antenna arrays is very often related with electronic beam steering. An important application of LPDA antennas is in air traffic, as it an essential part of localizer antenna array.
- A typical localizer antenna system is a part of the electronic systems known as *Instrumental Landing System* (ILS). Localizer shapes a radiation pattern providing lateral guidance to the aircraft beginning its descent, intercepting the projected runway center line, and then making a final approach.

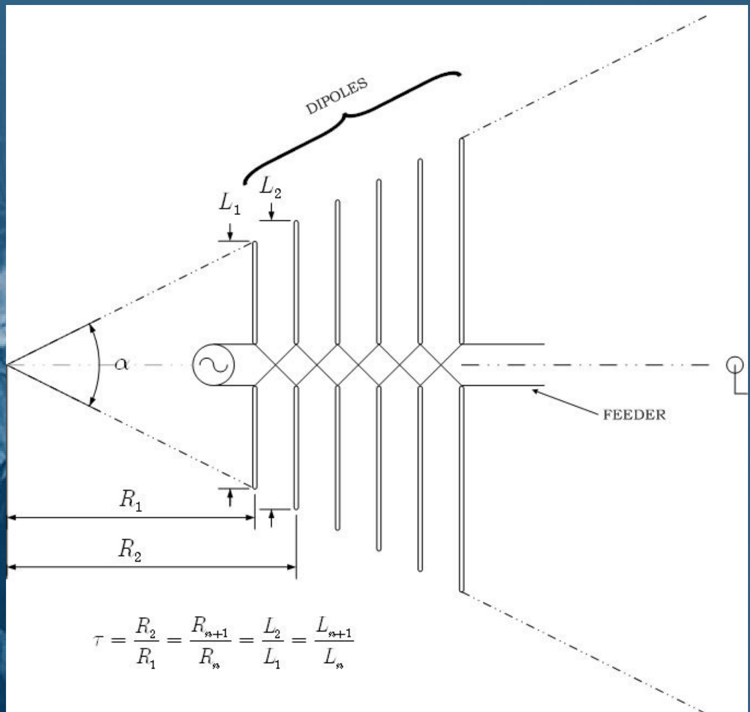
Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

The length of actual wire is obtained by multiplying the previous length and factor T:

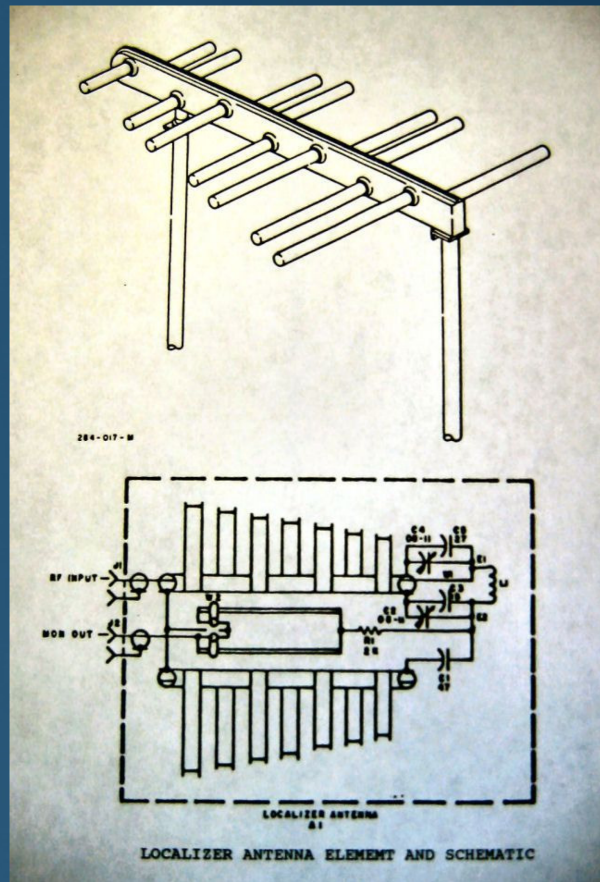
$$\tau = \frac{L_{n+1}}{L_n}$$

A look at a real localizer antenna element geometry...



Log-periodic dipole array

LPDA geometry



LOCALIZER ANTENNA ELEMEMT AND SCHEMATIC

FD analysis of wire antennas

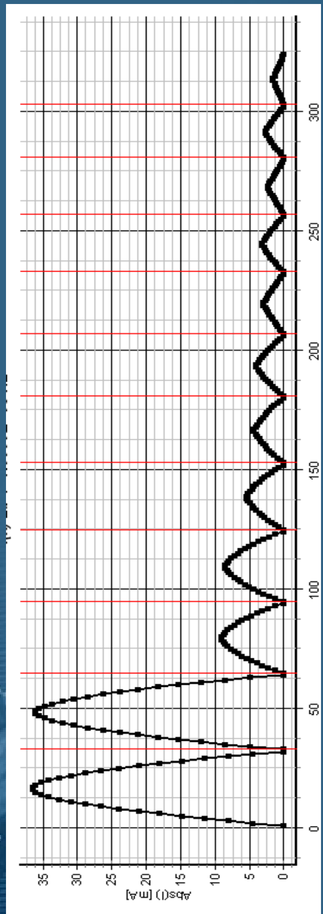
LPDA in free space

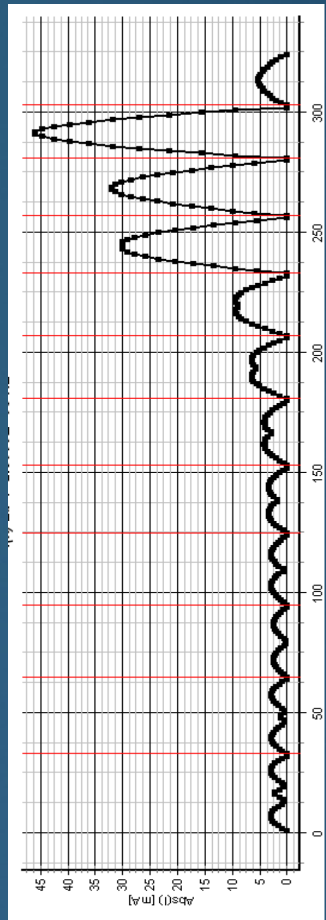
- LPDA is composed from 12 dipoles insulated in free space.
- The radius of all wires is a=0.004m while the length of wires are determined by the length of 1st wire L_1=1.5m, and factor T=0.9.
- All dipoles are fed by the voltage generator V_g=1V with variable phase (each time phase is changed for 180°).
- The operating frequency is varied from 100 MHz to 300 MHz.

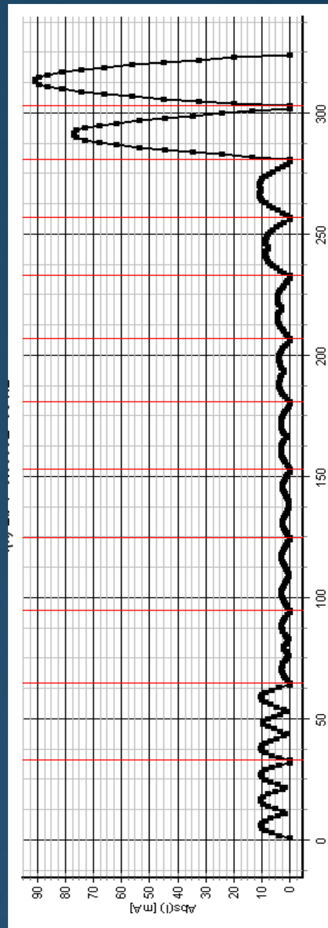
FD analysis of wire antennas

Department of Electronics
University of Split,
Split, Croatia

LPDA in free space







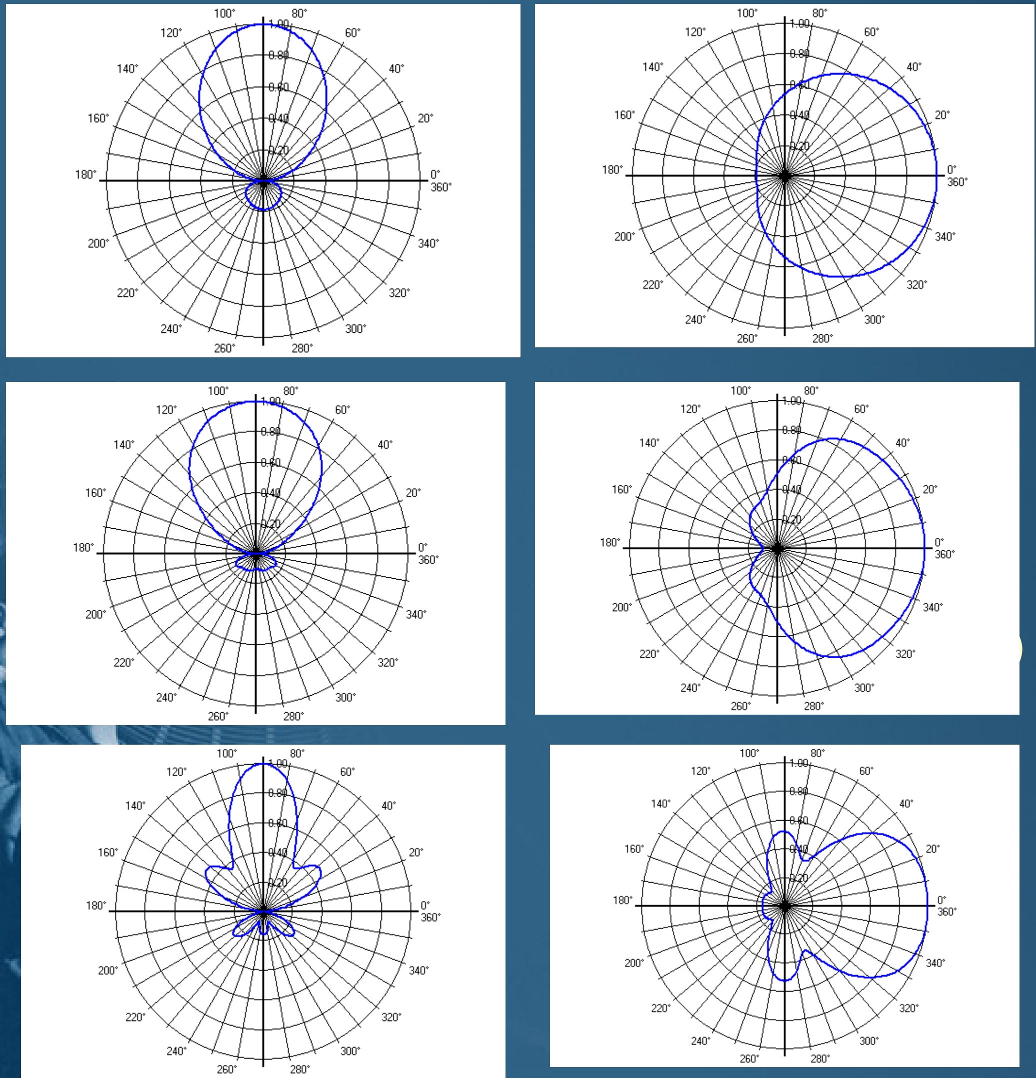
Absolute value of Current distribution along 12 dipoles versus BEM nodes at f=100MHz, f=250MHz and f=300MHz

FD analysis of wire antennas

Department of Electronics
University of Split,
Split, Croatia

LPDA in free space

Radiation pattern



f=100MHz

f=250MHz

f=300MHz

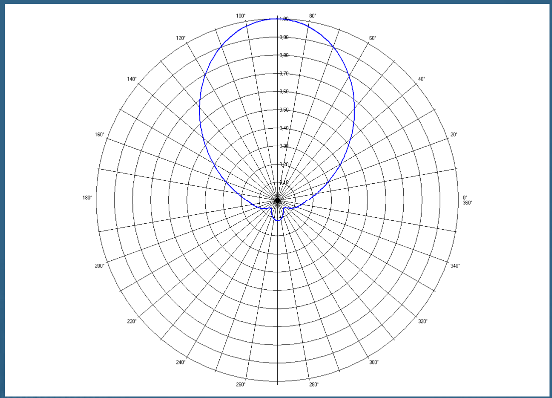
(XY plane) (YZ plane)

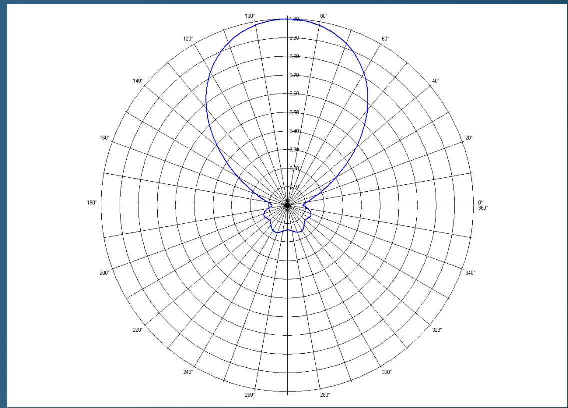
FD analysis of wire antennas

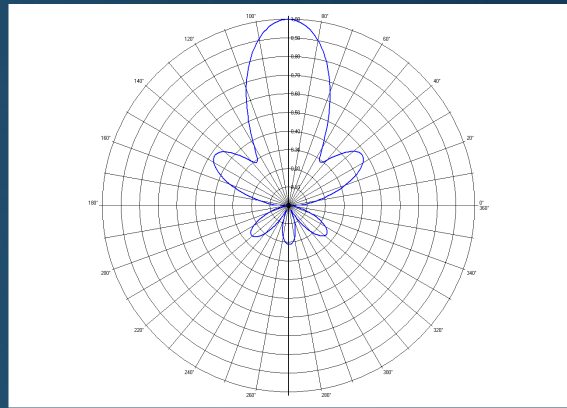
Department of Electronics
University of Split,
Split, Croatia

LPDA above a PEC ground

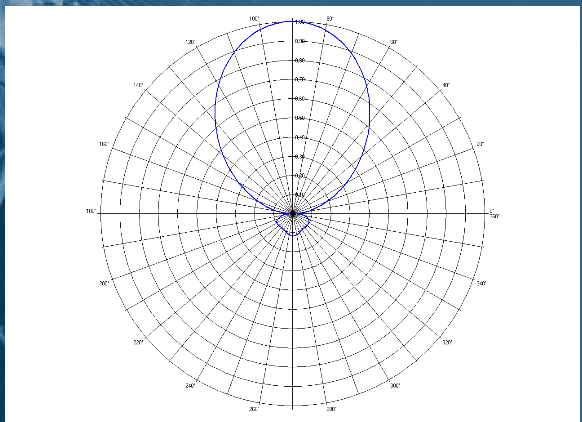
Radiation pattern (XY plane)

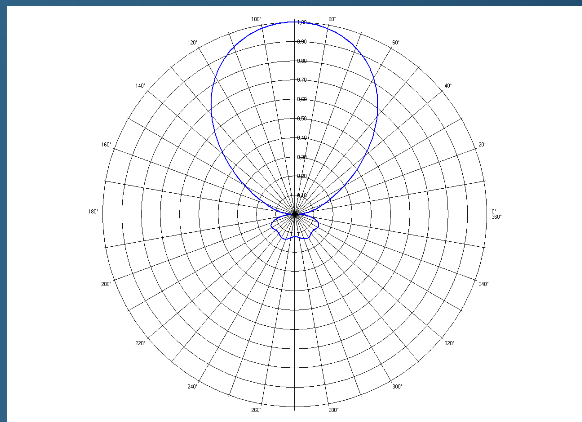


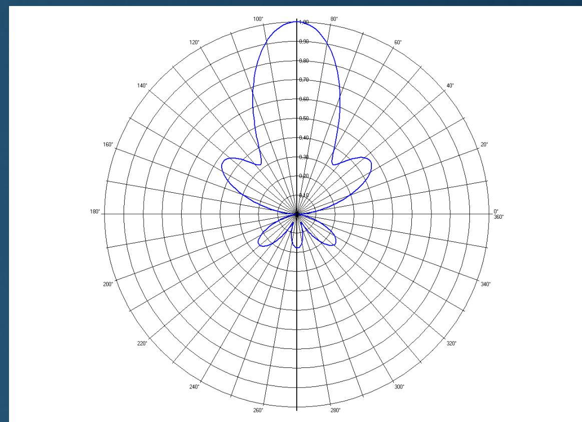




LPDA above a real ground







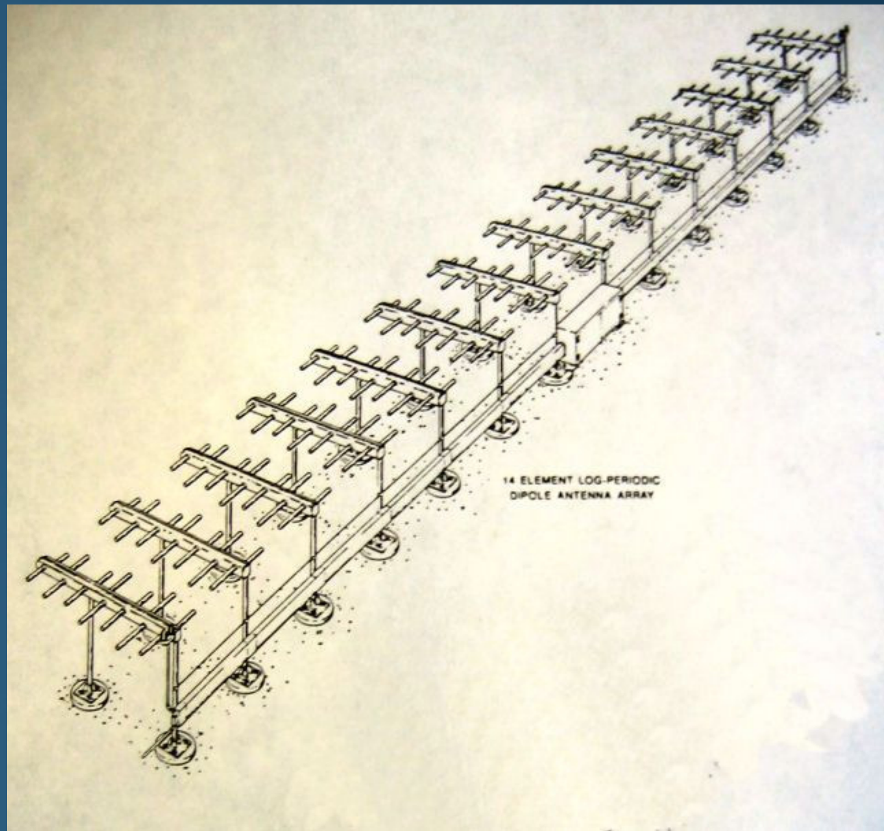
f=100MHz *f*=250MHz *f*=300MHz

FD analysis of wire antennas

Department of Electronics
University of Split,
Split, Croatia

- realistic geometries of localizer antenna systems.

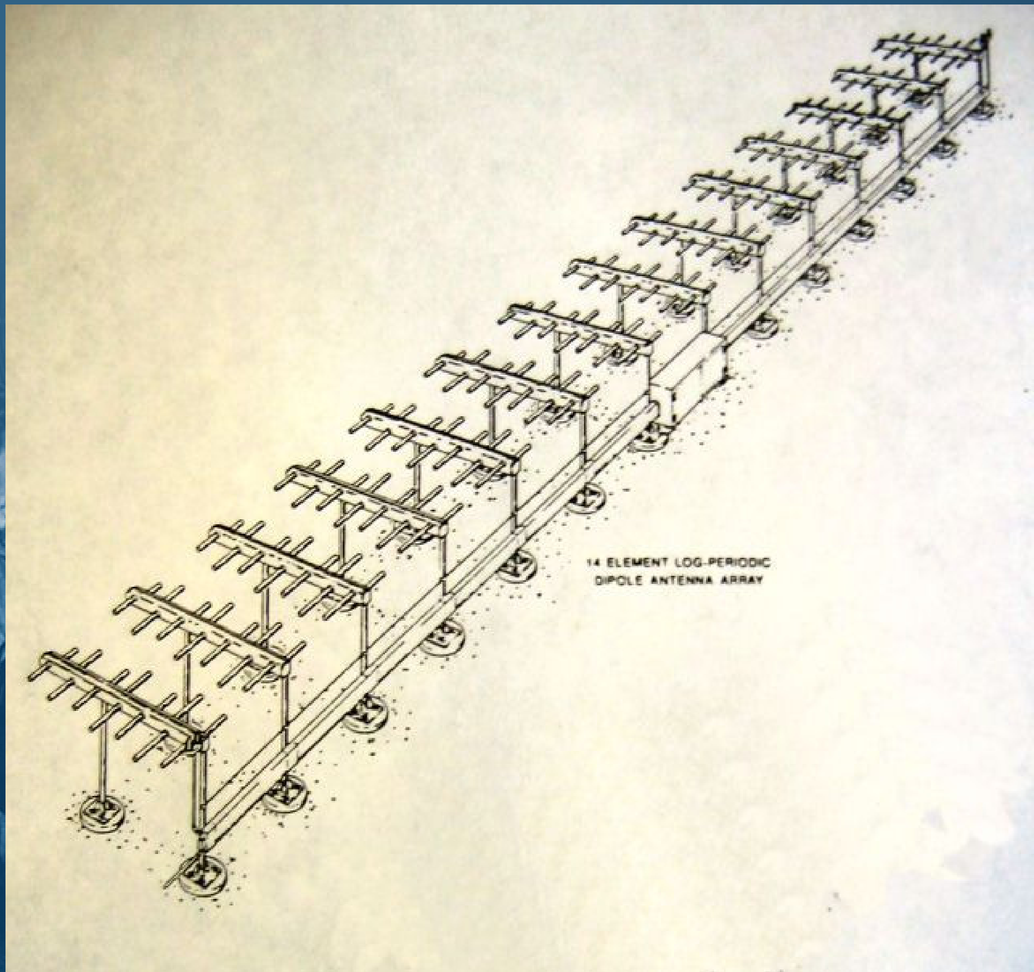
f=110MHz
T=0.983
σ=0.1876
L_1=1.27m
d_1=0.4765m
n=7 –wires per LPDA
a=0.002
N_{seg}=11 - segments per wire
N_{LPDA}=14
h=1.82m
σ=0.005
ε_r=13

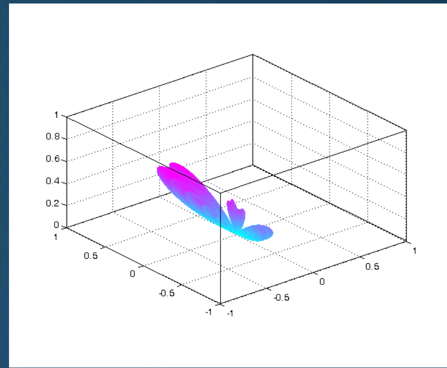


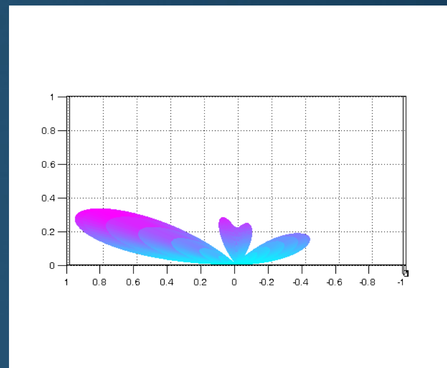
FD analysis of wire antennas

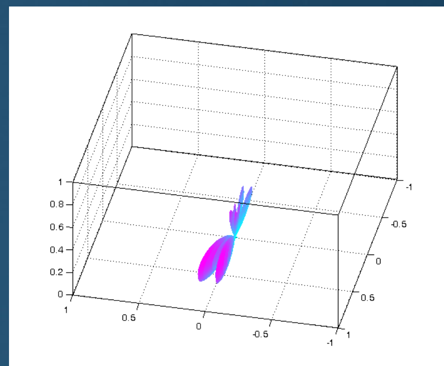
Department of Electronics
University of Split,
Split, Croatia

- realistic geometries of localizer antenna systems.





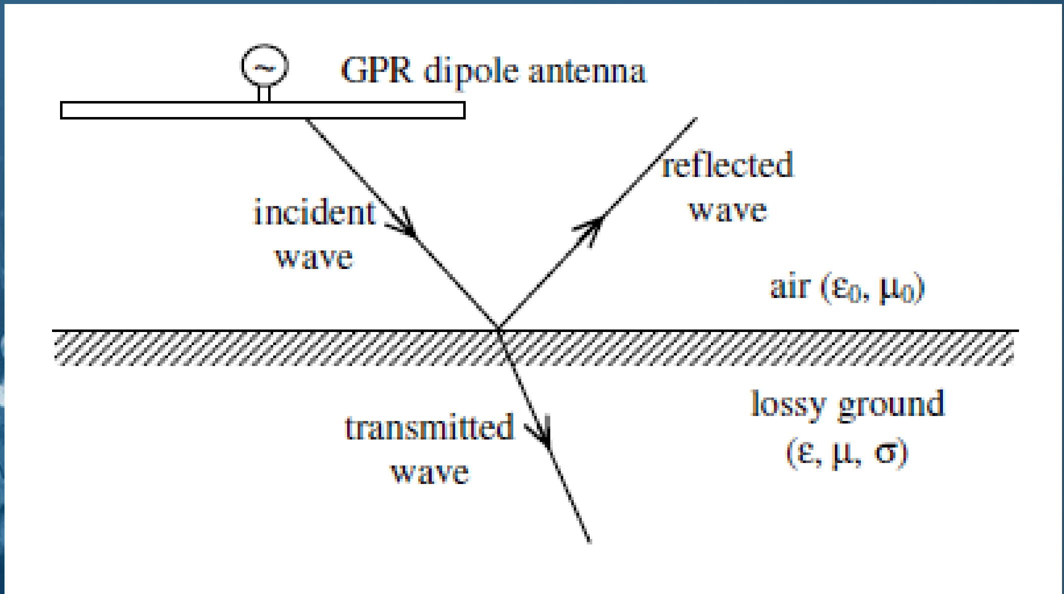




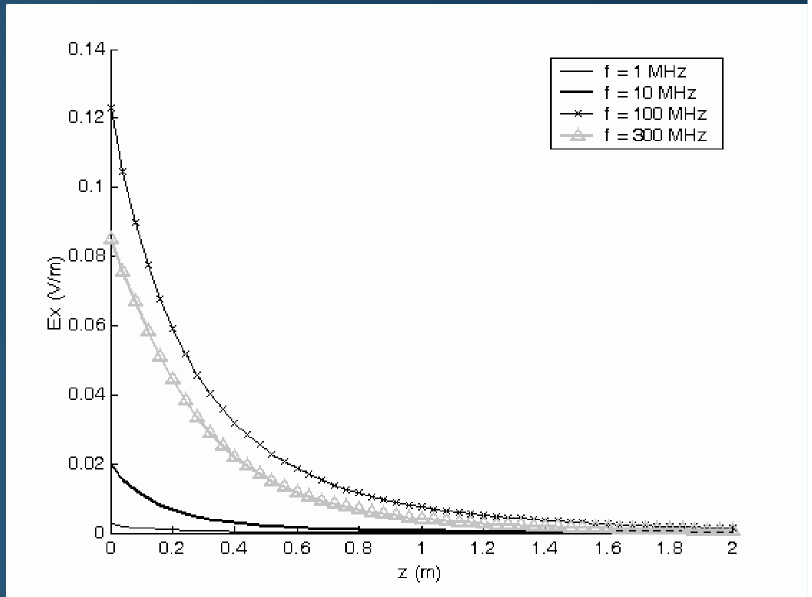
Department of Electronics
University of Split,
Split, Croatia

FD analysis of wire antennas

- Dipole antenna for Ground Penetrating Radar (GPR) applications



GPR dipole antenna above a lossy half-space



Broadside transmitted field (V/m) into the ground for different frequencies (L=1 m, a=2 mm, h=0.25 m, V_T=1 V, ε_{rg}=10, σ=10 mS/m)

Department of Electronics
University of Split,
Split, Croatia

Frequency domain analysis: Formulation

- Pocklington integro-differential equation:

$$E_x^{exc} = j\omega\frac{\mu}{4\pi}\int_{-L/2}^{L/2} I(x')g(x,x')\,dx' - \frac{1}{j4\pi\omega\varepsilon_0}\frac{\partial}{\partial x}\int_{-L/2}^{L/2}\frac{\partial I(x')}{\partial x'}g(x,x')\,dx'$$

- The transmitted electric field components:

$$E_x = \frac{1}{j4\pi\omega\varepsilon_{eff}}\left[-\int_{-L/2}^{L/2}\frac{\partial I(x')}{\partial x'}\frac{\partial G(x,x',z)}{\partial x'}dx' - \gamma^2\int_{-L/2}^{L/2} G(x,x',z)I(x')dx'\right]$$

$$E_z = \frac{1}{j4\pi\omega\varepsilon_{eff}}\int_{-L/2}^{L/2}\frac{\partial I(x')}{\partial x'}\frac{\partial G(x,x',z)}{\partial z}dx'$$

$$G(x,x') = \Gamma_{tr}^{MIT} g_E(x,x',z)$$

$$\Gamma_{tr}^{MIT} = \frac{2n}{n+1}$$

Department of Electronics
University of Split,
Split, Croatia

Frequency domain analysis: Numerical solution

- The current and its first derivative at the *i*-th boundary element are given by:

$$I(x') = I_{1i}\frac{x_{2i}-x'}{\Delta x} + I_{2i}\frac{x'-x_{1i}}{\Delta x}$$

$$\frac{\partial I(x')}{\partial x'} = \frac{I_{2i}-I_{1i}}{\Delta x}$$

Matrix equation:

$$\sum_{j=1}^{M}[Z]_{ji}\{I\}_i = \{V\}_j$$

Mutual impedance matrix, voltage vector:

$$[Z]_{ji} = -\frac{1}{j4\omega\pi\varepsilon_{eff}}\left[\int_{\Delta l_j}\{D\}_j\int_{\Delta l_i}\{D'\}_i^T g(x,x')\,dx'dx + \gamma^2\int_{\Delta l_j}\{f\}_j\int_{\Delta l_i}\{f'\}_i^T g(x,x')\,dx'dx\right]$$

$$\{V\}_j = -\int_{\Delta l_j} E_x^{inc}(x)\{f\}_j\,dx$$

Department of Electronics
University of Split,
Split, Croatia

Frequency domain analysis: Numerical solution

- The field formulas:

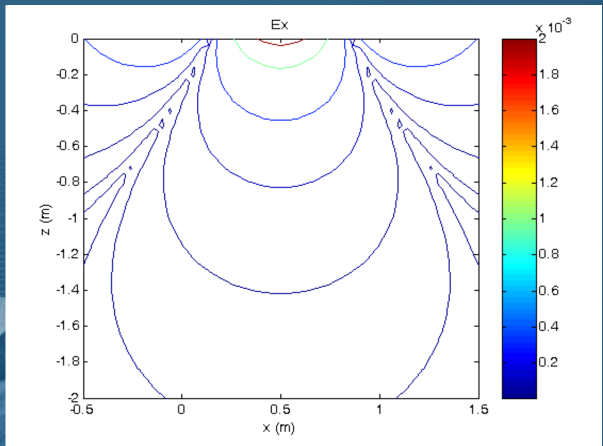
$$E_x = \frac{1}{j4\pi\omega\varepsilon_{eff}} \sum_{i=1}^{N_j} \left[-\frac{I_{2i} - I_{1i}}{\Delta x_j} \int_{x_{1ij}}^{x_{2i}} \frac{\partial G(x,x',z)}{\partial x'} dx' - \gamma^2 \int_{x_{1ij}}^{x_{2ij}} \left[I_{1i} \frac{x_{2i} - x'}{\Delta x} + I_{2i} \frac{x' - x_{1i}}{\Delta x} \right] G(x,x',z) I(x') dx' \right]$$

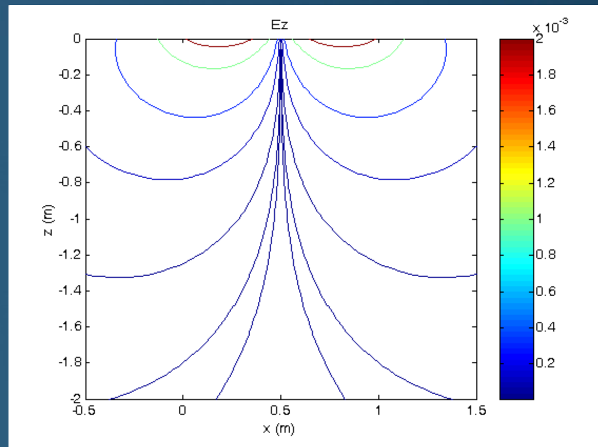
$$E_z = \frac{1}{j4\pi\omega\varepsilon_{eff}} \sum_{j=1}^{M} \sum_{i=1}^{N_j} \frac{I_{2ij} - I_{1ij}}{\Delta x_j} \int_{x_{1ij}}^{x_{2ij}} \frac{\partial G(x,x',z)}{\partial z} dx'$$

Department of Electronics
University of Split,
Split, Croatia

Frequency domain analysis: Numerical results

- The computational example; dipole antenna (L=1m, a=2mm, h=0,25m, ε_{rg}=10, σ=10mS/m).
- Terminal voltage is V_T=1V.
- The operating frequency: from 1MHz to 100MHz.



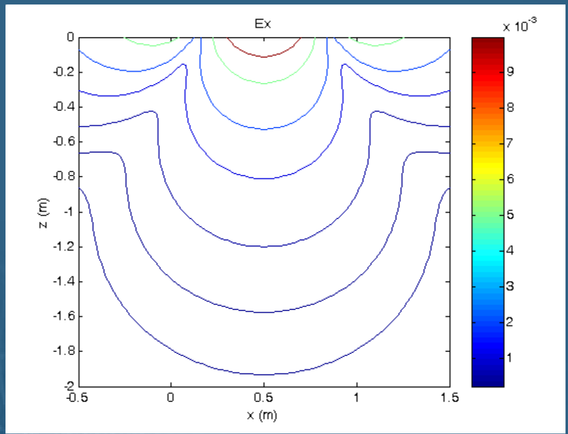


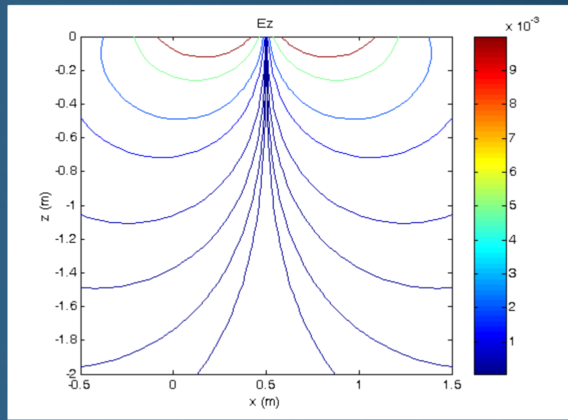
E_x – component E_z - component

Transmitted field (V/m) into the ground at f = 1MHz

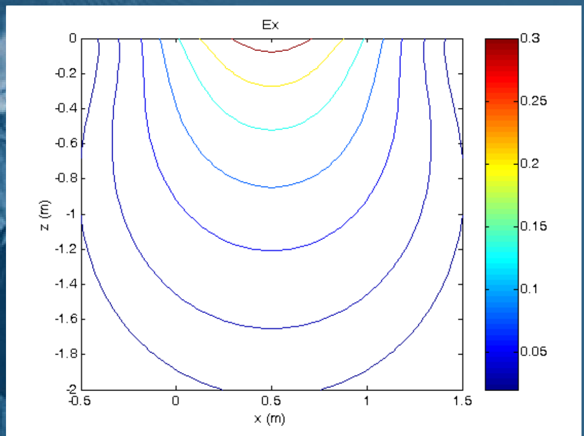
Department of Electronics
University of Split,
Split, Croatia

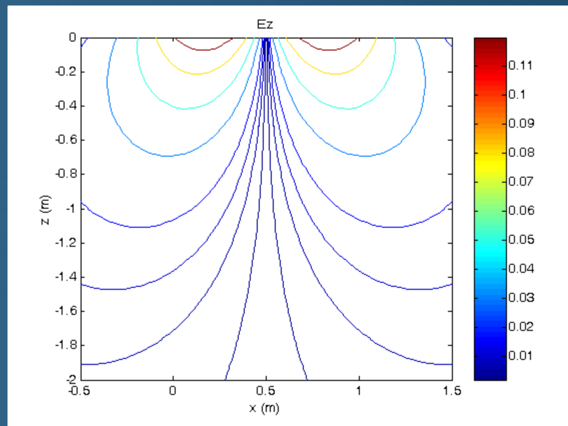
Frequency domain analysis: Numerical results





$f = 10\text{MHz}$





$f = 100\text{MHz}$

E_x – component

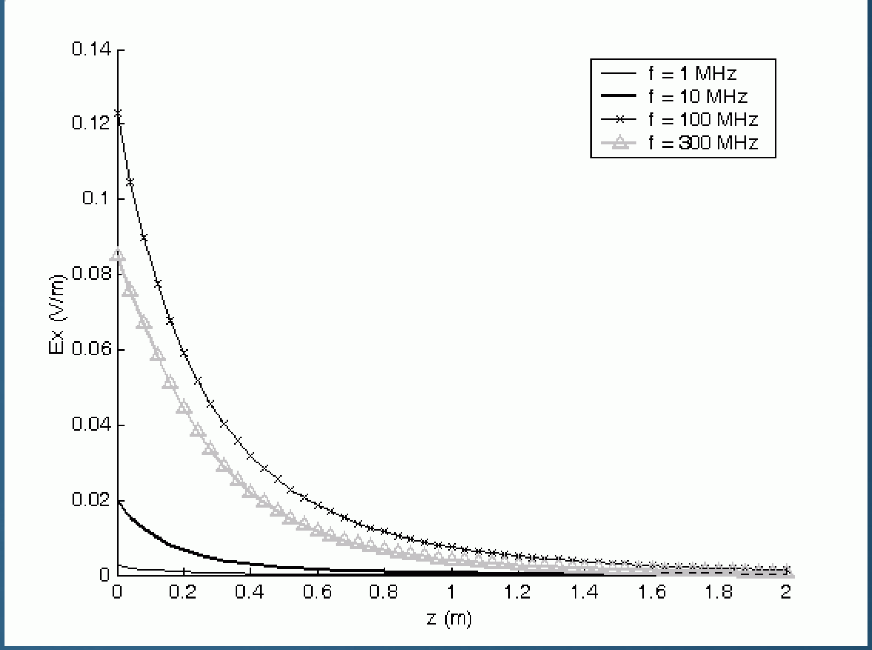
E_z - component

Transmitted field (V/m) into the ground

Department of Electronics
University of Split,
Split, Croatia

Frequency domain analysis: Numerical results

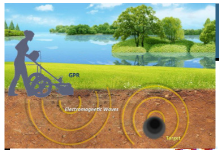
- E_x component of the transmitted field versus depth in the broadside direction for different operating frequencies



Broadside transmitted field (V/m) into the ground for different frequencies

Frequency domain analysis: Concluding remarks

- The FD analysis of *E*-field transmitted into the material half-space due to the GPR dipole antenna radiation is based on the Pocklington IDE and related field formulas.
- The influence of the earth-air interface is taken into account via the simplified reflection/transmission coefficient arising from the Modified Image Theory (MIT).
- The Pocklington IDE is solved via the Galerkin-Bubnov variant of the Indirect Boundary Element Method (GB-IBEM) and the corresponding transmitted field is determined by using BEM formalism, as well.



SUMMARY -SPECTRAL APPROACHES

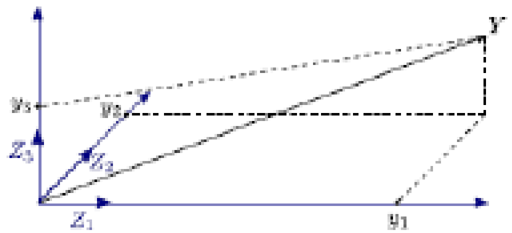
- Heuristic
  - Instead of considering the random output Y = M(X) through samples, Y is represented by a series expansion

$$Y = \sum_{j=0}^{+\infty} y_j \, Z_j$$

Sudret, *PCE Theory, Numerical Methods & Applications Parts I & II*, MNMUQ, 2014

where:

- $\{Z_j\}_{j=0}^{+\infty}$ is a numerable set of random variables that forms a basis of a suitable space $\mathcal{H} \supset Y$
- $\{y_j\}_{j=0}^{+\infty}$ is the set of coordinates of Y in this basis



- Actually
  - Truncated series expansion
  - EMC illustration: Y = current *I*

Random N-vector: $\mathbf{X} = (X_1, X_2, \ldots, X_N)$

Polynomial basis: $\Phi^u(\mathbf{X})$ function of $\mathbf{X}$

$$\boxed{[I]_u(\mathbf{X}) \approx \sum_{v_1=0}^{n_1} \cdots \sum_{v_N=0}^{n_N} \eta_w^{v_1 \ldots v_N} \, \Phi^u(\mathbf{X})}$$

Expansion coefficient: $\eta_w^{v_1 \ldots v_N}$

SUMMARY - PHILOSOPHY OF THE STOCHASTIC COLLOCATION

Basic idea: close to PCE (spectral method) $\rightarrow$ choice of polynomial basis

Finding a polynomial approximation of the function of a real variable

$$f(x) = \frac{1}{1+x^2}$$

Lagrange polynomials

$$f(x) \approx \sum_{i=0}^{n} f_i L_i(x)$$

$\{L_i(x)\}_{0 \le i \le n}$ nth order polynomial basis

$$L_i(x) = \prod_{\substack{j=0 \\ j \ne i}}^{n} \frac{x - x_j}{x_i - x_j}$$

We can demonstrate:

$$\boxed{f_i = f(x_i)}$$

Finding a polynomial approximation of the function of a random variable:

$$f(X) = \frac{1}{1+X^2}$$

Lagrange polynomials

$$f(X) \approx \sum_{i=0}^{n} f_i L_i(X)$$

$\Downarrow$

Stochastic collocation method

Problem:

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \qquad \boxed{Y = f(X)}$$

Bonnet et al., *Numerical simulation of a Reverberation Chamber with a stochastic collocation method, 2009*

SUMMARY - SC PRINCIPLE (1)

$\{x_j\}_{0\leq j\leq n}$ defined by the Gauss quadrature rule

$$I=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}f(x)dx\approx\sum_{j=0}^{n}\omega_j f(x_j)$$

Mean value assessment

$$<Y>=<f(X)>=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}f(x)dx$$

Replacing f(x) by its expansion

$$\omega_i=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}L_i(x)dx$$

$$<Y>=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\left(\sum_{i=0}^{n}f_iL_i(x)\right)dx=\sum_{i=0}^{n}f_i\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}L_i(x)dx \Rightarrow I_i=\sum_{j=0}^{n}\omega_jL_i(x_j)$$

$L_i(x_j)=\delta_{ij}$ $\Longrightarrow$ All the terms are equal to 0 except at its particular collocation point i:

$$<Y>\approx\sum_{i=0}^{n}\omega_i f_i$$

Variance assessment

$$\mathrm{var}(Y)=\sum_{k=0}^{n}\omega_k(f_k)^2-<Y>^2$$

SUMMARY - SC PRINCIPLE (2)

$$E(Z^0;t)=\sum_{i=0}^{n}E_i(Z^0)L_i(t) \qquad L_i(t_j)=\delta_{ij} \qquad E_i(Z^0)=E(Z^0;t_i) \qquad \int_D pdf(u)f(u)du=\sum_{i=0}^{n}\omega_i f(t_i)$$

Mean value derivation

$$\langle E(Z^0;t)\rangle=\int_D E(Z^0;u)\,pdf(u)du \qquad \langle E(Z^0;t)\rangle=\sum_{i=0}^{n}E_i\left(Z^0\right)\int_D L_i(u)\,pdf(u)du=\sum_{i=0}^{n}\omega_i E_i\left(Z^0\right)$$

Variance derivation

$$\sigma^2=\int_D\left[E(Z^0;u)-\langle E(Z^0;t)\rangle\right]^2 pdf(u)du \qquad \sigma^2=\int_D\left[\sum_{i=0}^{n}E_i(Z^0)L_i(u)-\sum_{i=0}^{n}\omega_i E_i(Z^0)\right]^2 pdf(u)du$$

$$\sigma^2=\int_D E^2(Z^0,u)\,pdf(u)du-2\int_D E(Z^0,u)\langle E(Z^0,t)\rangle\,pdf(u)du+\langle E(Z^0,t)\rangle^2\int_D pdf(u)du$$

$$\sigma^2=\int_D E^2(Z^0,u)\,pdf(u)du-2\langle E(Z^0,t)\rangle\int_D E(Z^0,u)\,pdf(u)du+\langle E(Z^0,t)\rangle^2$$

$$\sigma^2=\int_D E^2(Z^0,u)\,pdf(u)du-\langle E(Z^0,t)\rangle^2=\langle E^2(Z^0,t)\rangle-\langle E(Z^0,t)\rangle^2$$

$$\sigma^2=\sum_{i=0}^{n}\omega_i E_i^2(Z^0)-\left(\sum_{i=0}^{n}\omega_i E_i(Z^0)\right)^2 \qquad \sigma^2=\langle E^2(Z^0,t)\rangle-\langle E(Z^0,t)\rangle^2$$

SUMMARY - SC PRINCIPLE (3)

- Determination of weights w_i and points x_i
- Computation of the system response

$$f_i = f(x_i) = \frac{1}{1 + x_i^2}$$

- Mean value and variance assessment methods are given by:

$$< f(X) >= \sum_{i=0}^{n} \omega_i f_i$$

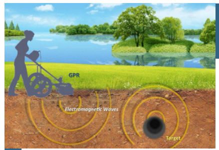
$$\mathrm{var}(f) = \sum_{i=0}^{n} \omega_i f_i^2 - < f(X) >^2$$

N=n+1	Weights	Points
2	0.5000 0.5000	1 -1
3	0.1667 0.6667 0.1667	1.7321 0 -1.7321
4	0.0459 0.4541 0.4541 0.0459	2.3344 0.7420 -0.7420 -2.3344
5	0.0113 0.2221 0.5333 0.2221 0.0113	2.8570 1.3556 0 -1.3556 -2.8570

Normal distribution (Gauss-Hermite)

Small number N of collocation points !

General approach



SUMMARY - SC PRINCIPLE (4)

- Random parameter: $Z \equiv \hat{u} = Z^0 + \hat{u}^0$
  Z^0 central value, $\hat{u}^0$ Random Variable (RV) arbitrarily given
- Uniform, normal, exponential ... laws ($\hat{u}$)
- Stat. moments from output I computed from "$n+1$ well chosen" weighted (ω_i) points I_i [3]

Mean $\langle I \rangle = \sum_{i=0}^{n} \omega_i I_i$	Variance $\sigma_I^2 = \sum_{i=0}^{n} \omega_i I_i^2 - \langle I \rangle^2$

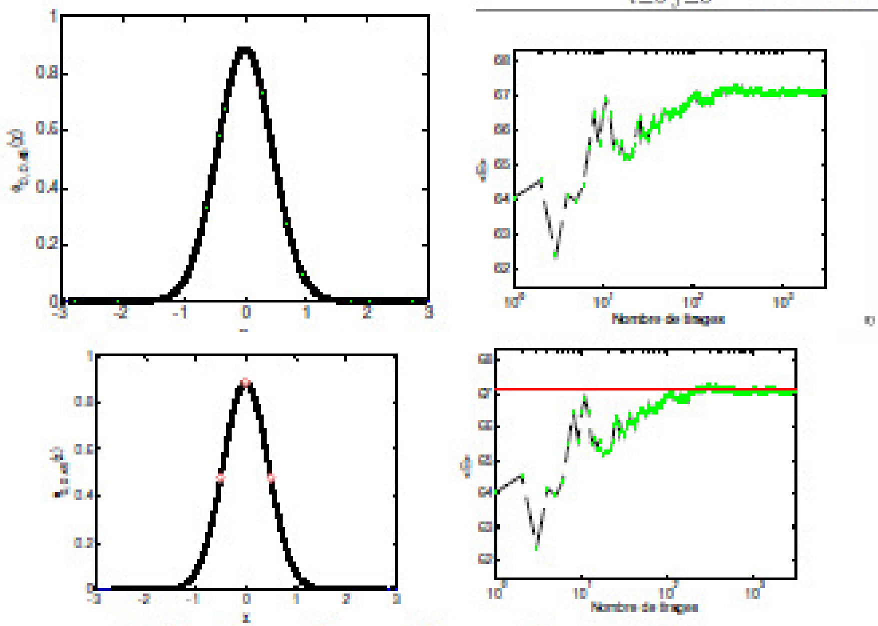
Extension to multi-RV

Statistical moment	Computation
1st	$mean(I) = \sum_{i=0}^{n1} \sum_{j=0}^{n2} \omega_i^1 \omega_j^2 I_{ij}$
2nd	$variance(I) = \sum_{i=0}^{n1} \sum_{j=0}^{n2} \omega_i^1 \omega_j^2 I_{ij}^2 - [mean(I)]^2$
3rd	$skewness(I) = \sum_{i=0}^{n1} \sum_{j=0}^{n2} \frac{[\omega_i^1 \omega_j^2 I_{ij} - mean(I)]^3}{[variance(I)]^{\frac{3}{2}}}$
4th	$kurtosis(I) = \sum_{i=0}^{n1} \sum_{j=0}^{n2} \frac{[\omega_i^1 \omega_j^2 I_{ij} - mean(I)]^4}{[variance(I)]^2}$

Modeling « Uncertainties »

Validation

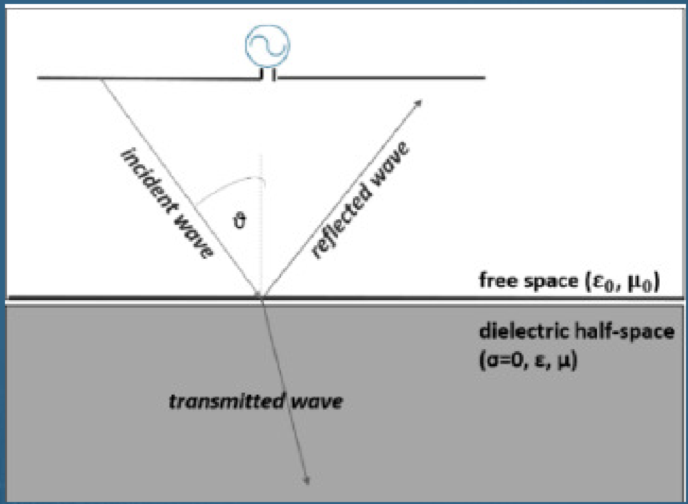
SC = smart MC



COST TU 1208 Split, Croatia, 7 November 2016

Stochastic Modeling : Numerical results

Department of Electronics
University of Split,
Split, Croatia



The horizontal dipole antenna placed above the dielectric half-space has length L=1 m and radius r=6.74 mm. Both length and radius of the antenna are considered as deterministic parameters.

Two parameters are modelled as random input variables following the uniform distributions: the height of the antenna: $h \sim \cup(0.275, 0.725)$ m and soil permittivity $\varepsilon_r \sim \cup(4, 30)$.

$$V(f) = V_0 \cdot \sqrt{\pi}\, t_w\, e^{-(\pi f t_w)^2}\, e^{-j2\pi f t_0}$$

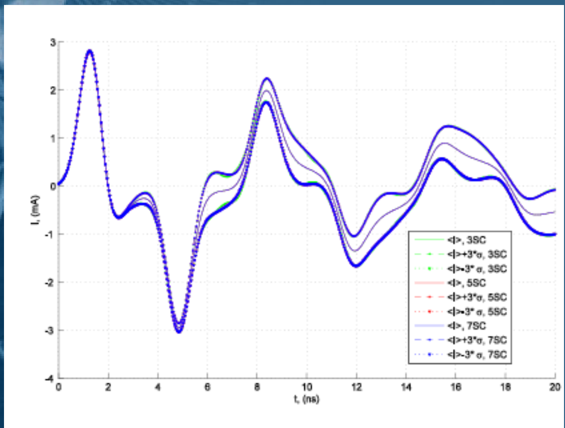


Fig. 4 Current at the center of the wire with antenna height as random variable: $\langle I \rangle$ is stochastic expected value and σ_i is standard deviation (univariate example No. 2)

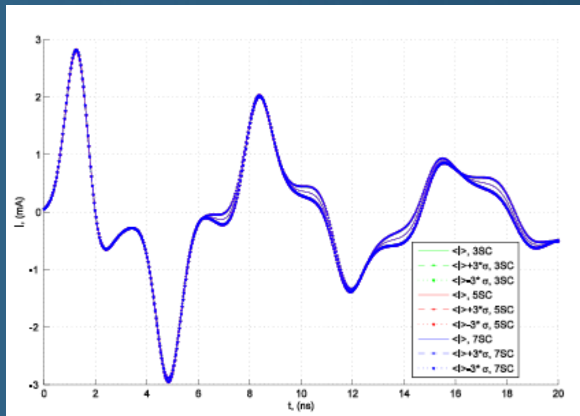


Fig. 2 Current at the center of the wire with soil permittivity as random variable: $\langle I \rangle$ is stochastic expected value and σ_i is standard deviation (univariate example No. 1)

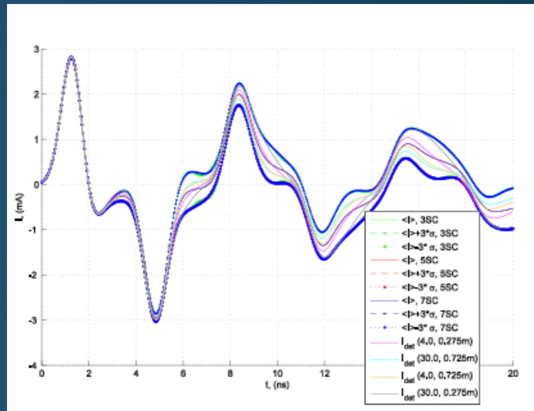
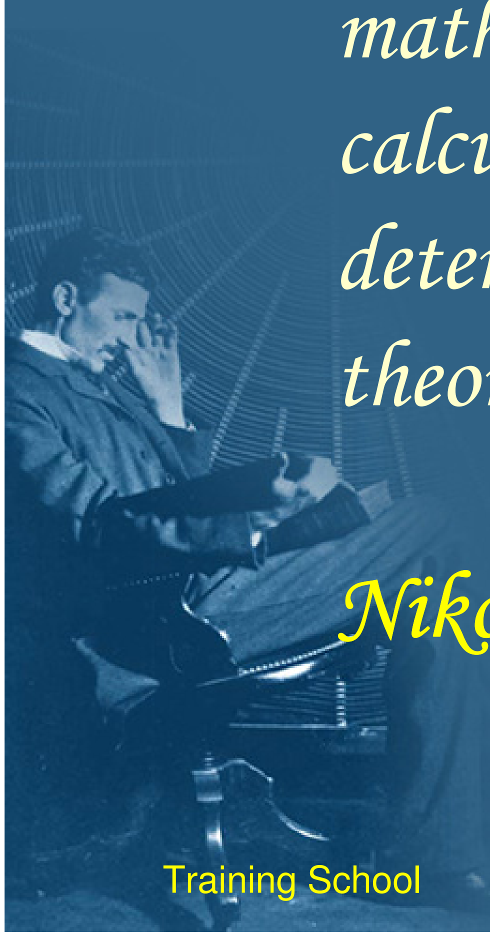


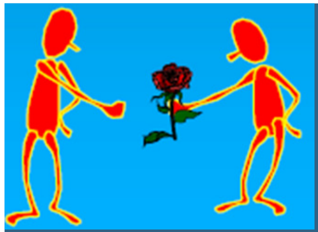
Fig. 6 Current at the center of the wire with two random input variables: soil permittivity and antenna height: $\langle I \rangle$ is stochastic expected value and σ_i is standard deviation (multivariate example)

Department of Electronics
University of Split,
Split, Croatia

There is scarcely a subject that cannot be mathematically treated and the effect calculated beforehand, or the results determined beforehand from the available theoretical and practical data.



Nikola Tesla



Department of Electronics
University of Split,
Split, Croatia

We made models in science, but we also made them in everyday life.
STEPHEN HAWKING

Thank you for your attention