COST Action TU1208
"Civil Engineering Applications of Ground Penetrating Radar"

Introduction to electromagnetic modelling of GPR & overview on available commercial and free software tools

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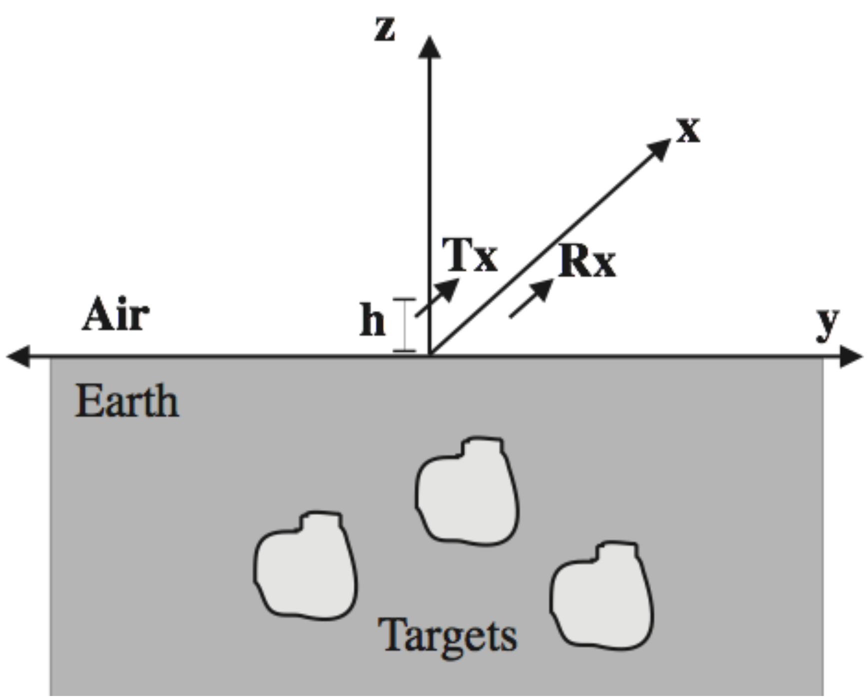
EM Modelling for GPR – What do we model?

- Knowing the initial and boundary conditions (properties of the electromagnetic source; geometrical and physical features of the scenario), the goal is to find a solution to the *GPR forward scattering problem* and calculate the electromagnetic field in any point of the space and in any time instant.



EM Modelling for GPR – Why?

- **To improve our understanding of GPR**

GPR radargrams often have no resemblance to the subsurface or structures over which the profile was recorded. Various factors, including the innate design of the survey equipment and the complexity of electromagnetic propagation in the ground/structure, can disguise complex structures recorded on GPR reflection profiles.

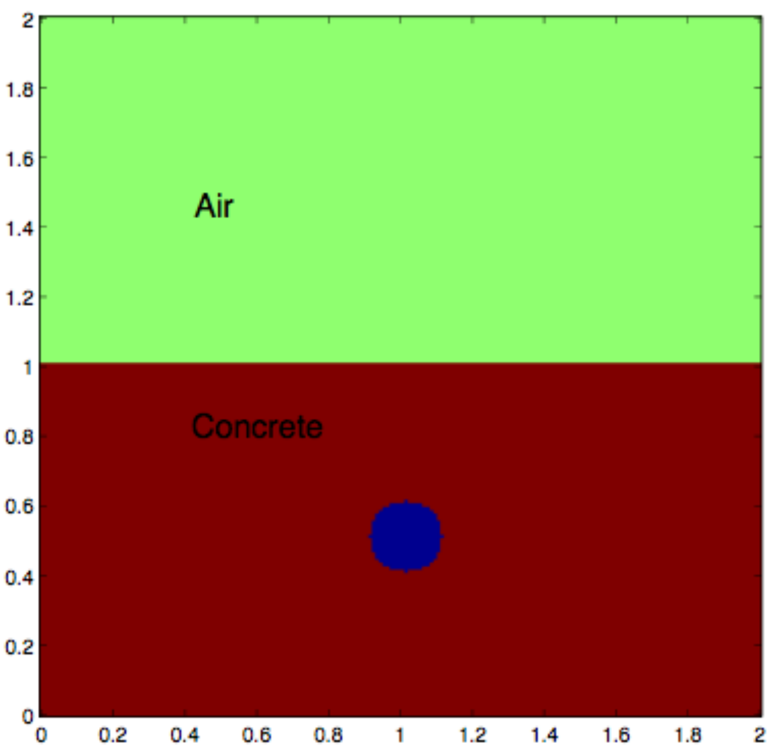
An EM simulator can help us to understand how target structures get translated into GPR radargrams. It can also show the limitations of GPR, highlight its capabilities and help us to understand where and in what environments this technique can be effectively used.

EM Modelling for GPR – Why?

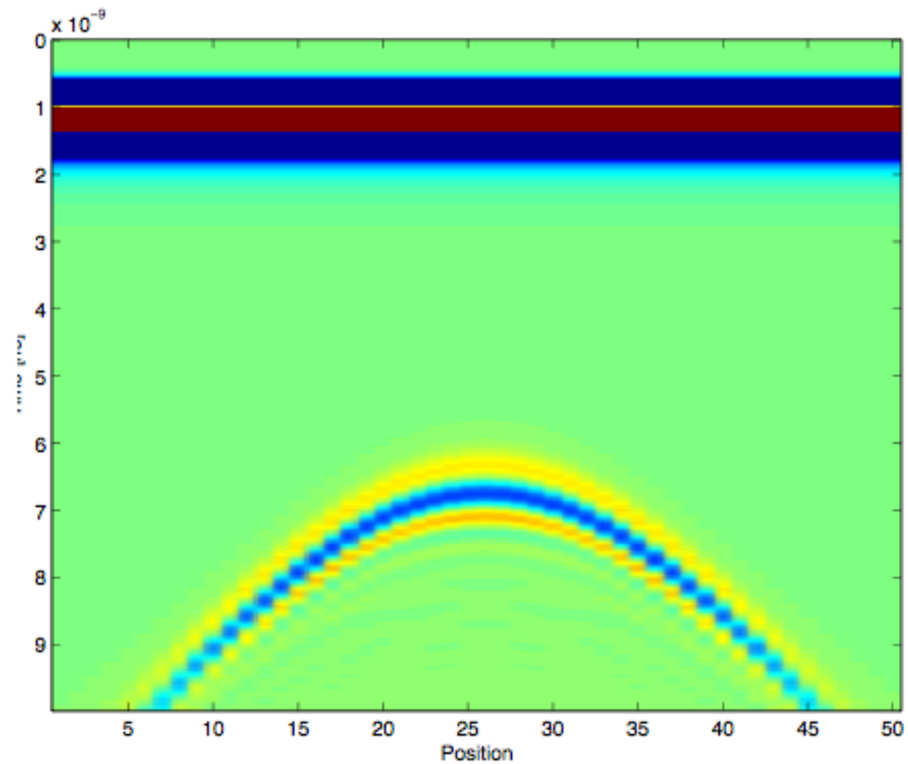
- **To choose the most proper GPR equipment for a survey.**
- **To aid the interpretation of complex data sets** and find out unknown details about the investigated structure/subsurface.
- **To test new data-processing/imaging algorithms** or assess the effectiveness of existing ones.
- **To design and optimise new GPR antennas.**
- **A forward EM solver can be used as part of an inverse EM solver** (automatic interpretation of GPR data).

EM Modelling for GPR – Why?

Geometry



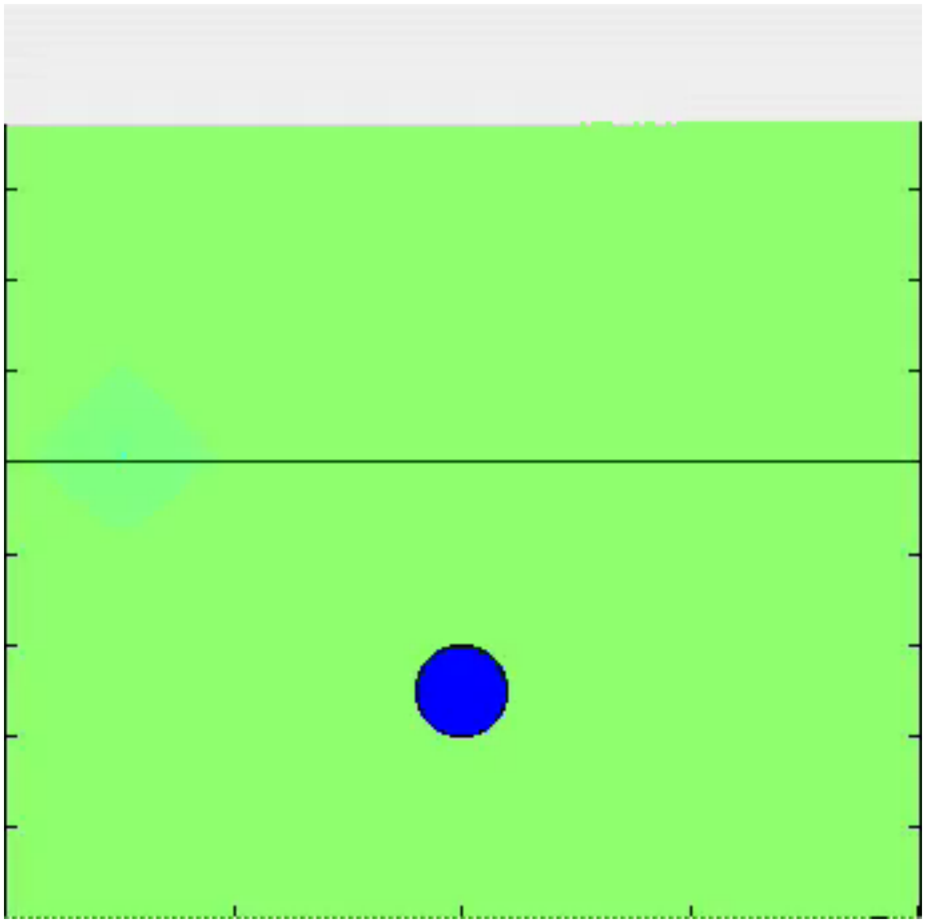
Synthetic Radargram



This is a simple scenario, the main purpose of an electromagnetic simulator is not to solve such a simple problem …

EM Modelling for GPR – Why?

Field distribution as a function of time



… but still, this movie effectively shows how the electromagnetic simulator can help us to visualize and understand the electromagnetic phenomena taking place in a GPR scenario.

EM Modelling for GPR – How can we do it?

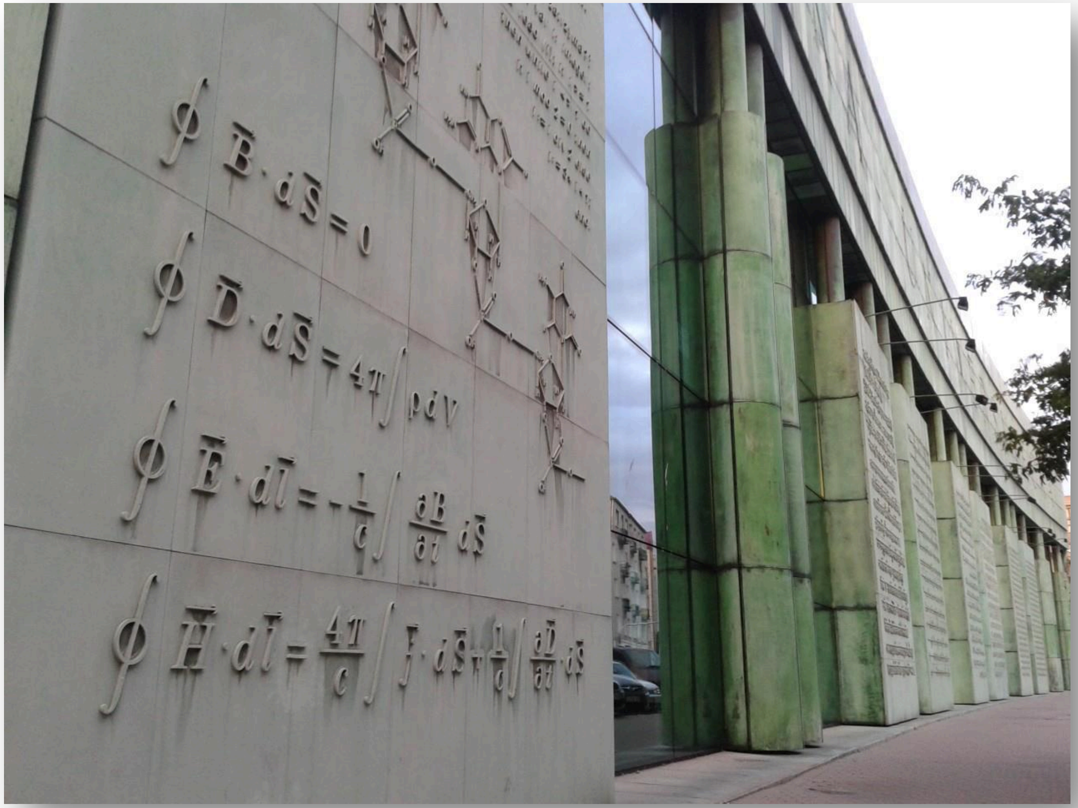
- By solving the **Maxwell's equations** governing the propagation and scattering of electromagnetic waves, subject to initial and boundary conditions.

André-Marie Ampère
(1775-1836)

Michael Faraday
(1791-1867)

Carl Friedrich Gauss
(1777-1853)

James Clerk Maxwell
(1831-1879)



Wall of the Warsaw University Library

EM Modelling for GPR – Maxwell's Equations

Faraday's Law of induction

$$\oint_C \mathbf{E}\cdot d\hat{\mathbf{l}} = -\iint_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\hat{\mathbf{s}}$$

describes how a time varying magnetic field creates an electric field

Ampere-Maxwell Law

$$\oint_C \mathbf{H}\cdot d\hat{\mathbf{l}} = \iint_S \frac{\partial \mathbf{D}}{\partial t}\cdot d\hat{\mathbf{s}} + \iint_S \mathbf{J_c}\cdot d\hat{\mathbf{s}} + \iint_S \mathbf{J_s}\cdot d\hat{\mathbf{s}}$$

states that magnetic fields can be generated in two ways: by an electrical current (the original "Ampère's law") and by changing electric fields ("Maxwell's addition")

EM Modelling for GPR – Maxwell's Equations

Gauss' Electric and Magnetic Laws

$$\oiint_S \mathbf{D} \cdot d\hat{\mathbf{s}} = \iiint_V q dV$$

describes the relationship between the electric field and the electric charges that cause it

$$\oiint_S \mathbf{B} \cdot d\hat{\mathbf{s}} = 0$$

states that there are no "magnetic charges"

Continuity of Electric Charge (not part of Maxwell's equations)

$$\oiint_S \mathbf{J} \cdot d\hat{\mathbf{s}} = -\iiint_V \frac{\partial q}{\partial t} dV$$

an empirical law expressing (local) charge conservation; mathematically it is an automatic consequence of Maxwell's eqs.; it states that the divergence of the current density J is equal to the negative rate of change of the charge density ρ

EM Modelling for GPR – Maxwell's Equations

Integral equations

$$\oint_C \mathbf{E} \cdot d\hat{\mathbf{l}} = -\iint_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\hat{\mathbf{s}}$$

$$\oint_C \mathbf{H} \cdot d\hat{\mathbf{l}} = \iint_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\hat{\mathbf{s}} + \iint_S \mathbf{J_c} \cdot d\hat{\mathbf{s}} + \iint_S \mathbf{J_s} \cdot d\hat{\mathbf{s}}$$

$$\oiint_S \mathbf{D} \cdot d\hat{\mathbf{s}} = \iiint_V q dV \qquad \oiint_S \mathbf{B} \cdot d\hat{\mathbf{s}} = 0$$

EM Modelling for GPR – Maxwell's Equations

By using Stoke's and divergence theorems…

$$\oint_S \mathbf{A} \cdot d\hat{\mathbf{l}} = \iint_S (\nabla \times \mathbf{A}) \cdot d\hat{\mathbf{s}}$$

$$\oiint_S \mathbf{A} \cdot d\hat{\mathbf{s}} = \iiint_V (\nabla \cdot \mathbf{A}) dV$$

…we get the differential equations

EM Modelling for GPR – Maxwell's Equations

Differential Equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J_c} + \mathbf{J_s}$$

$$\nabla \cdot \mathbf{D} = q_v \qquad \nabla \cdot \mathbf{B} = 0$$

These are first order partial differential equations which express the relations between the fundamental electromagnetic field quantities and their dependence on their sources.

EM Modelling for GPR – Maxwell's Equations

- In Maxwell's equations, the field vectors are assumed to be single-valued, bounded, continuous functions of position and time.

- In order to simulate the GPR response from a particular target or set of targets, the integral or differential Maxwell's equations have to be solved subject to the geometry of the problem and the initial conditions.

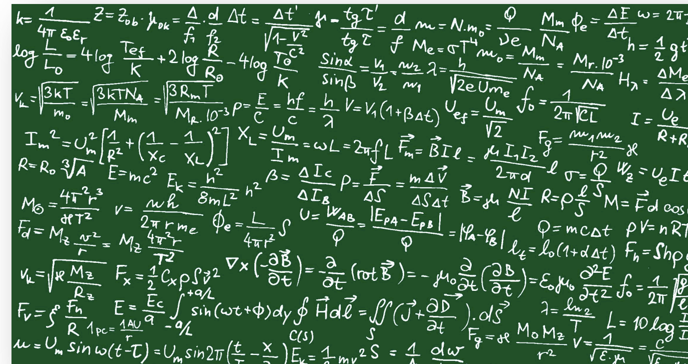
EM Modelling for GPR – Constitutive Relations

$$
\begin{aligned}
\mathbf{D} &= \bar{\bar{\epsilon}} * \mathbf{E} \\
\mathbf{B} &= \bar{\bar{\mu}} * \mathbf{H} \\
\mathbf{J_c} &= \bar{\bar{\sigma}} * \mathbf{E}
\end{aligned}
$$

- For simple cases where the electrical properties can be assumed to be frequency independent, the convolutions reduce to multiplications. For isotropic media, the tensors reduce to scalars. In these cases, computations are simplified.

$$
\begin{aligned}
\mathbf{D} &= \epsilon \mathbf{E} \\
\mathbf{B} &= \mu \mathbf{H} \\
\mathbf{J_c} &= \sigma \mathbf{E}
\end{aligned}
\qquad \epsilon_r = \frac{\epsilon}{\epsilon_0} \qquad \mu_r = \frac{\mu}{\mu_0}
$$

EM Modelling for GPR – How can we solve Maxwell's equations?



- **Analytically** with great brain effort and only for simple scenarios!

    Lots of brain power

    Little computing power

- **Numerically** with less brain effort and for more complex and realistic scenarios!

    Lots of computing power

    Little brain power

$\int \partial$

Integral / Differential Formulations

Frequency / Time Domain

$\partial / \partial t$

$j\omega$

EM Modelling for GPR – How can we solve Maxwell's equations?

Computational issues

- **Accuracy**
- **Execution time**
- **Memory requirements**

- Higher accuracy → longer time & higher memory requirements.
- Analytical-numerical techniques contemplating a relatively simple mathematical formulation and a mostly numerical nature are easy to implement and versatile.
- To develop and implement full-wave formulations with higher analytical complexity, a deeper physical insight into the considered problem is needed. Usually, these approaches are less versatile and, when applicable, they are fast and efficient.

EM Modelling for GPR – What is the best computational method?

- Method of Moments (MoM)
- Boundary Element Method (BEM)
- Finite Element Method (FEM)
- Finite Difference Frequency Domain (FDFD)
- Partial Element Equivalent Circuit (PEEC)
- Finite Integration Technique (FIT)
- Asymptotic Methods (GTD/UTD/PO)
- Finite Difference Time Domain (FDTD)
- Transmission Line Matrix Method (TLM)
- Finite Element Time Domain (FETD)
- Finite Volume Time Domain (FVTD)
- Generalized Multipole Technique (GMT)
- Time-Domain Method of Moments (TDMoM)
- Cylindrical and Spherical Wave Approach (CWA and SWA)
- Other Methods

EM Modelling for GPR – Commercial software

- Ansoft HFSS
- CST Microwave Studio
- COMSOL Multiphysics
- XFDTD
- …

A good list is at
www.cvel.clemson.edu/modeling/EMAG/csoft.html

EM Modelling for GPR – Freeware tools

A good list is at
http://www.cvel.clemson.edu/modeling/EMAG/free-codes.html

- **NEC2** - the Numerical Electromagnetics Modeling (NEC) code is a widely used 3D simulator based on the method of moments. It was developed at Lawrence Livermore National Laboratory more than 10 years ago and has been compiled and run on many different computer systems. It uses a text interface. There a several free or inexpensive graphical interfaces that do pre- and post-processing of NEC2 models. A good free interface is 4nec2.

- There is a 2D FDTD solver included in **matGPR** (freeware tool for GPR data processing)

EM Modelling for GPR – Freeware tools

- **MEEP** – a FDTD simulation software package developed at MIT. It supports 1D/2D/3D/cylindrical problems, distributed-memory parallelism, and dispersive and nonlinear media. PML boundaries are implemented. The software is completely scriptable via both C ++ and Scheme (GNU Guile) interfaces.

- **GprMax** – a FDTD solver, for 2D (TMz)/3D numerical modelling. The first version was developed by Prof. Antonis Giannopoulos (The University of Edinburgh, UK) in 1996. In the past 3 years, gprMax has been significantly improved by the research team of The University of Edinburgh and as a contribution to the Action TU1208. **E^2GPR** is a free graphical interface developed by a research team of Sapienza/Roma Tre University, IT, to do pre- and post-processing of gprMax models.

www.gprmax.com
www.GPRadar.eu

Thank you!

Join COST Action TU1208!

www.GPRadar.eu

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