COST Action TU1208

"Civil Engineering Applications of Ground Penetrating Radar"

Cylindrical Wave Approach (CWA) for electromagnetic modelling of 2D GPR scenarios

Lara Pajewski

Sapienza University, Rome, Italy

lara.pajewski@uniroma1.it

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Talk Outline

- Theoretical Method
- Numerical Solution
- Results and Applications
- Conclusions and Work in Progress

CWA: Theoretical Method

CWA: Analytical-numerical technique for the solution of the direct two-dimensional scattering problem by a finite set of buried cylinders

Source

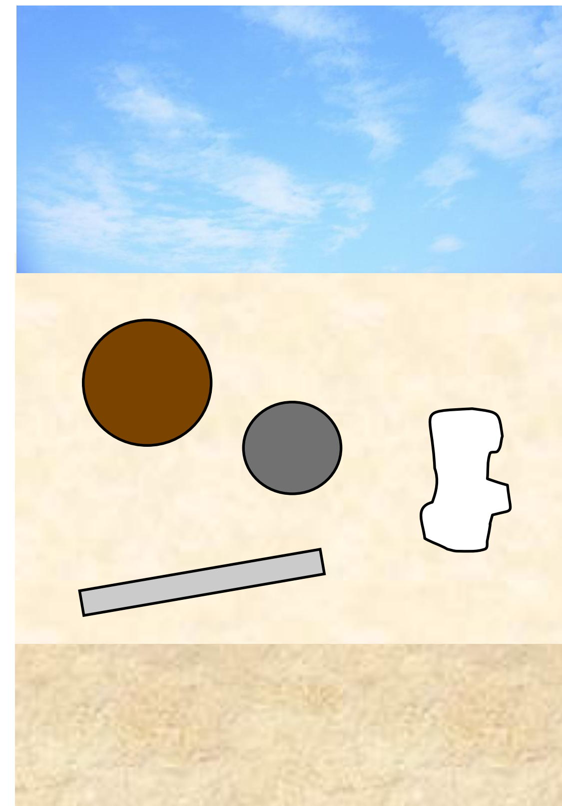
- Monochromatic or pulsed plane-wave
- Line sources of current or arbitrary 2D field distribution

Host medium

- Linear, isotropic, dielectric, half-space or finite-thickness slab

Buried Objects

- Perfectly-conducting or dielectric circular cylinders
- Scatterers with arbitrary cross-section, simulated by a suitable configuration of circular cylinders



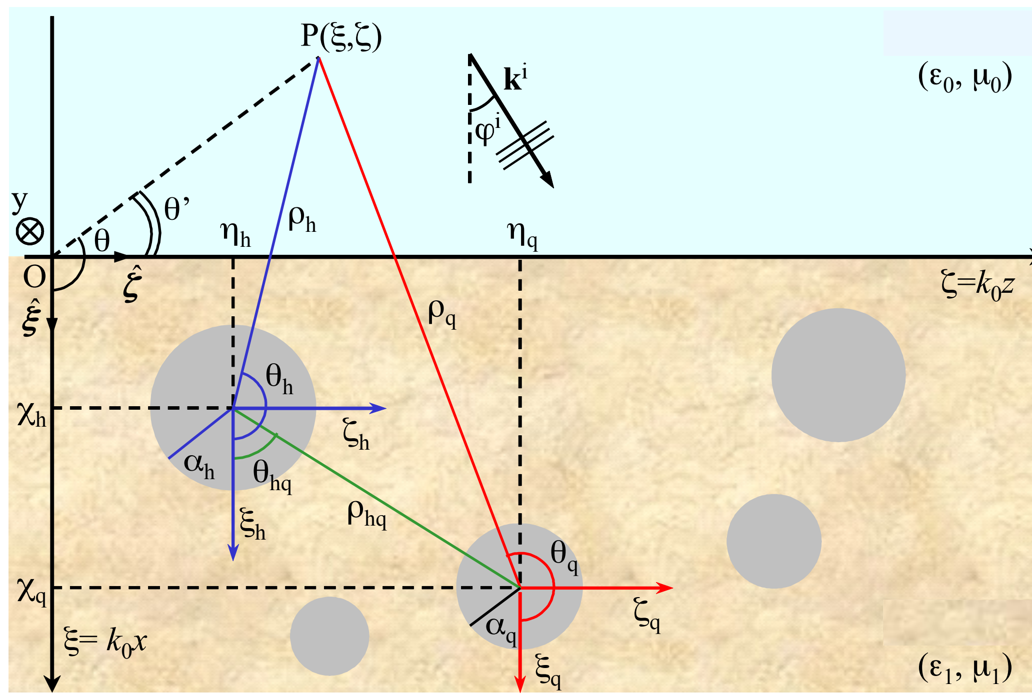
Lossless materials in this presentation,
Nicola will explain how to deal with losses!

CWA: Theoretical Method

Monochromatic plane-wave scattering problem by N perfectly-conducting circular cylinders buried in a dielectric half-space

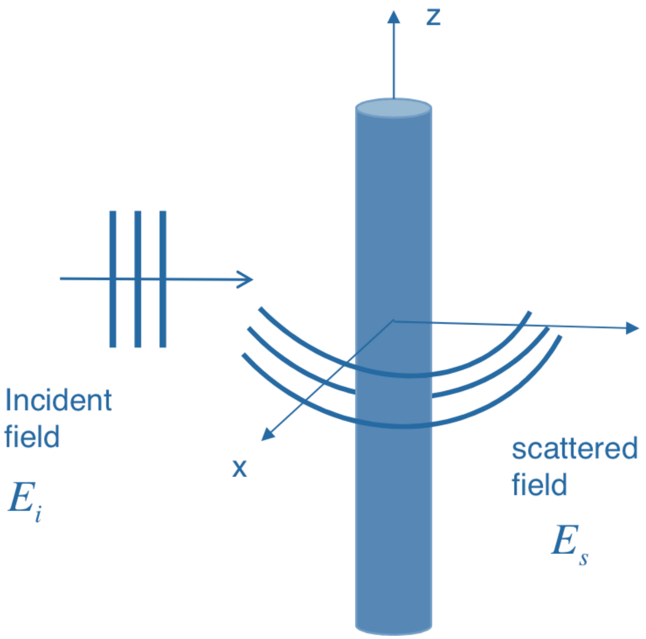
(IEEE Trans. Antennas and Propagation, 53(2), 2005, pp. 719-727)

- Arbitrary radii, burial depths, and distances between the obstacles
- All the cylinder-cylinder interactions, and the multiple reflections between the cylinders and the interface, are considered
- Results in near- and far-field zones, E and H polarization

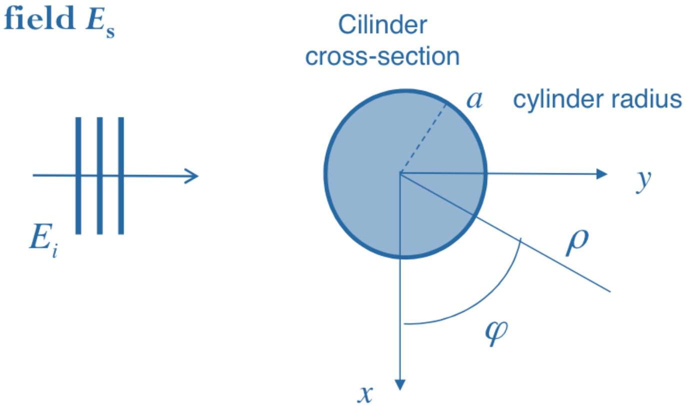


CWA: Theoretical Method

The scattering problem of a plane wave impinging on a circular-section cylinder, with infinite length and in free space, can be solved analytically with an expansion of the electromagnetic field into cylindrical waves.



Scattered field E_s



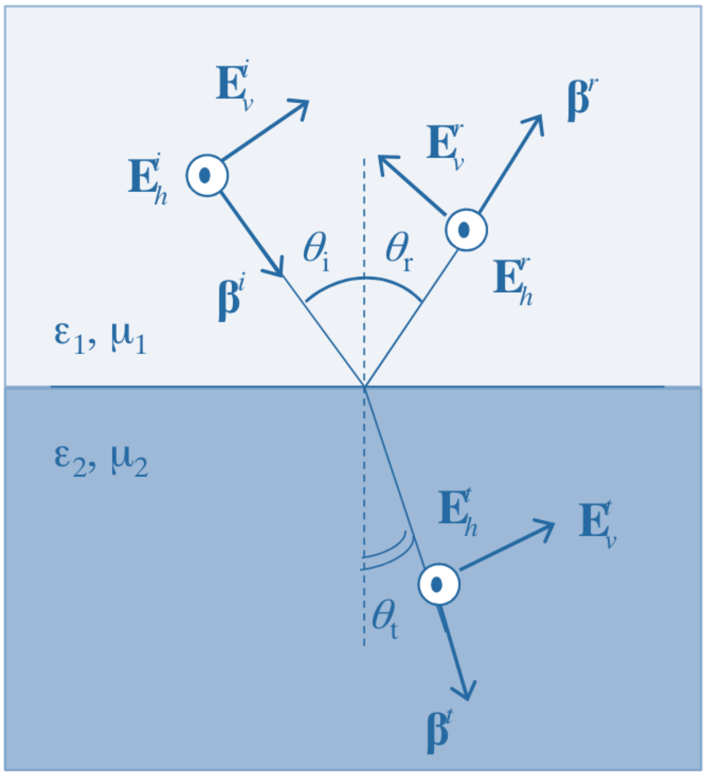
$$E_s(x,z) = E_0 \sum_{m=-\infty}^{+\infty} c_m H_m^{(1)}(k_0 \rho)$$

$H_m^{(1)}(k\rho)$ **First kind Hankel function of order m: a cylindrical function**

$$c_m = -i^{-m} \frac{H_m^{(1)}(k_0 a)}{H_m^{(1)}(k_0 a)} e^{in\varphi}$$ **Expansion coefficients (TM polarization)**

CWA: Theoretical Method

The interaction of a plane wave with a flat interface separating two half-spaces can be solved analytically by using the Fresnel coefficents and the Snell's law.



$$\mathbf{r} = x\mathbf{x}_0 + y\mathbf{y}_0 + z\mathbf{z}_0$$

$$\varepsilon_1 = \varepsilon_0 \varepsilon_{r1} \qquad \varepsilon_2 = \varepsilon_0 \varepsilon_{r2}$$

$$\mu_1 = \varepsilon_0 \mu_{r1} \qquad \mu_2 = \varepsilon_0 \mu_{r2}$$

Incident field

$$\mathbf{E}^i = E_h^i e^{-i\boldsymbol{\beta}_i \cdot \mathbf{r}} \mathbf{h}_0 + E_v^i e^{-i\boldsymbol{\beta}_i \cdot \mathbf{r}} \mathbf{v}_0$$

Reflected field

$$\mathbf{E}^r = \Gamma_h E_h^i e^{-i\boldsymbol{\beta}_r \cdot \mathbf{r}} \mathbf{h}_0 + \Gamma_v E_v^i e^{-i\boldsymbol{\beta}_r \cdot \mathbf{r}} \mathbf{v}_0$$

Transmitted field

$$\mathbf{E}^t = T_h E_h^i e^{-i\boldsymbol{\beta}_t \cdot \mathbf{r}} \mathbf{h}_0 + T_v E_v^i e^{-i\boldsymbol{\beta}_t \cdot \mathbf{r}} \mathbf{v}_0$$

CWA: Theoretical Method

The Fresnel coefficents give the amplitude of the reflected and transmitted plane waves.

REFLECTION COEFFICIENTS

- **Horizontal polarization**

$$\Gamma_h = \frac{\sqrt{(\mu_2 / \varepsilon_2)}\cos\theta_i - \sqrt{(\mu_1 / \varepsilon_1)}\cos\theta_t}{\sqrt{(\mu_2 / \varepsilon_2)}\cos\theta_i + \sqrt{(\mu_1 / \varepsilon_1)}\cos\theta_t}$$

- **Vertical polarization**

$$\Gamma_v = \frac{\sqrt{(\mu_1 / \varepsilon_1)}\cos\theta_i - \sqrt{(\mu_2 / \varepsilon_2)}\cos\theta_t}{\sqrt{(\mu_1 / \varepsilon_1)}\cos\theta_i - \sqrt{(\mu_2 / \varepsilon_2)}\cos\theta_t}$$

TRANSMISSION COEFFICIENTS

- **Horizontal polarization**

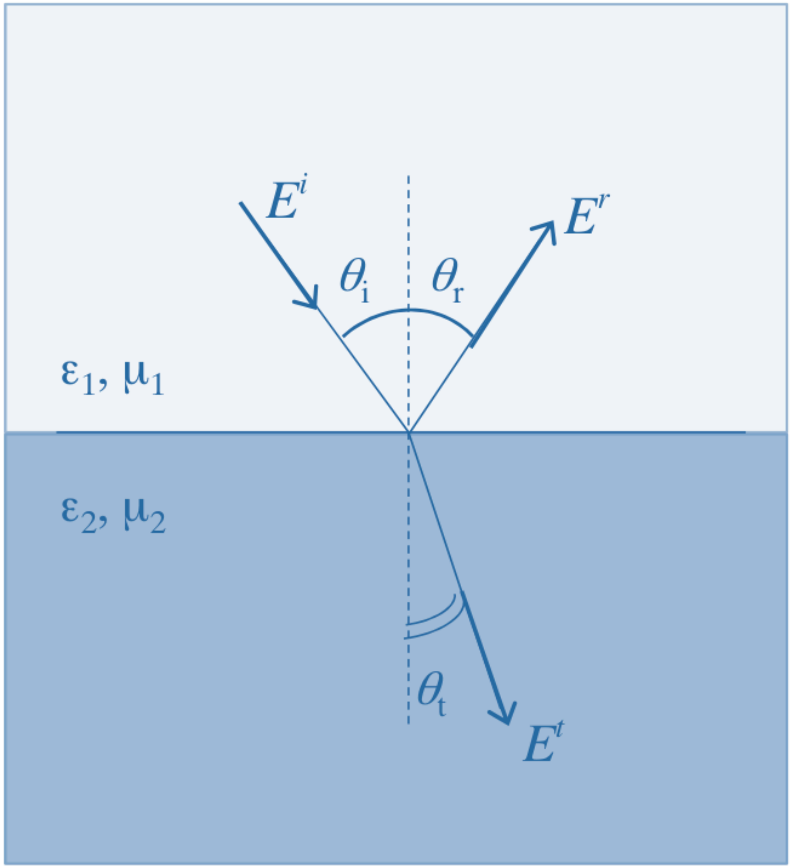
$$T_h = 1 - \Gamma_h$$

- **Vertical polarization**

$$T_v = 1 - \Gamma_v$$

CWA: Theoretical Method

The Snell's law gives the propagation direction of the reflected and transmitted plane waves.



- **Angle of reflection**

$$\theta_r = \theta_i$$

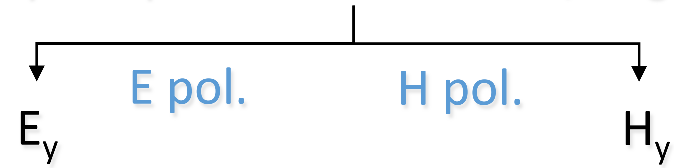
- **Angle of transmission**

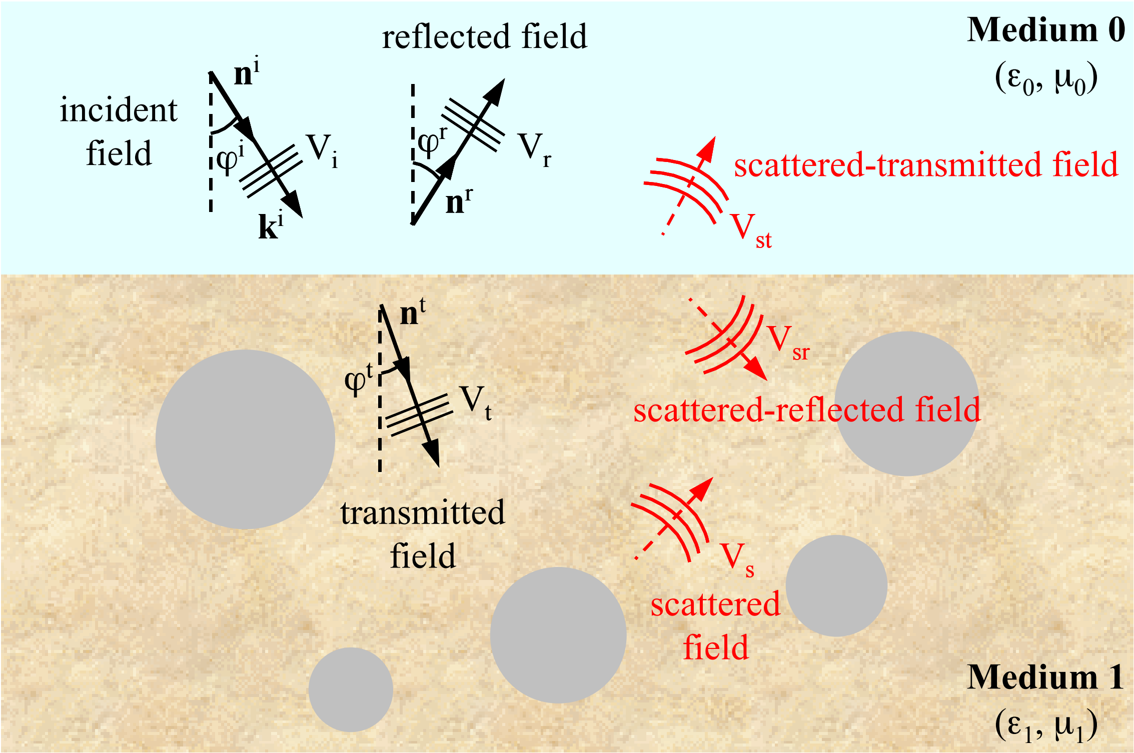
$$\sqrt{\varepsilon_{r1}}\sin\theta_i = \sqrt{\varepsilon_{r2}}\sin\theta_t$$

$$\Rightarrow \theta_t = \arcsin\left(\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\sin\theta_i\right)$$

CWA: Theoretical Method

V(x,z,t): y-component of the electric/magnetic field





CWA: Theoretical Method

- Scattered field represented as a superposition of cylindrical waves
- Plane-wave spectrum to take into account the reflection and transmission of cylindrical waves by the interface

Cylindrical Function

$$CW_{\mathrm{m}}(\xi,\zeta) = H_{\mathrm{m}}^{(1)}(\rho)\, e^{im\theta} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_{\mathrm{m}}(\xi, n_{||})\, e^{i n_{||}\zeta}\, dn_{||}$$

$F_{\mathrm{m}}(\xi, n_{||})$ → plane-wave spectrum

Reflected Cylindrical Function

$$RW_{\mathrm{m}}(\xi,\zeta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Gamma(n_{||}) F_{\mathrm{m}}(\xi, n_{||})\, e^{i n_{||}\zeta}\, dn_{||}$$

$\Gamma(n_{||})$ → plane-wave reflection coefficient

Transmitted Cylindrical Function

$$TW_{\mathrm{m}}(\xi,\zeta,\chi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} T_{10}(n_{||}) F_{\mathrm{m}}(-n_1\chi, n_{||})\, e^{i n_1 n_{||}\zeta}\, e^{-i(\xi+\chi)\sqrt{1-(n_1 n_{||})^2}}\, dn_{||}$$

$T_{10}(n_{||})$ → plane-wave transmission coefficient

CWA: Theoretical Method

Scattered Field

$$V_s(\xi,\zeta) = V_0 \sum_{\ell=-\infty}^{+\infty} J_\ell(n_1\rho_s) e^{i\ell\theta_s} \sum_{q=1}^{N} \sum_{m=-\infty}^{+\infty} i^m e^{-im\varphi_t} c_{qm}$$

$$\times \left[CW_{m-\ell}(n_1\xi_{qs}, n_1\zeta_{qs})(1-\delta_{qs}) + \frac{H_\ell^{(1)}(n_1\rho_s)}{J_\ell(n_1\rho_s)} \delta_{qs}\delta_{\ell m} \right]$$

unknown coefficients

Scattered-Reflected Field

$$V_{sr}(\xi,\zeta) = V_0 \sum_{\ell=-\infty}^{+\infty} J_\ell(n_1\rho_s) e^{i\ell\theta_s} \sum_{q=1}^{N} \sum_{m=-\infty}^{+\infty} c_{qm} i^m e^{-im\varphi_t}$$

$$\times RW_{m+\ell}\left[-n_1(\chi_q+\chi_s), n_1(\eta_s-\eta_q)\right]$$

Scattered-Transmitted Field

$$V_{st}(\xi,\zeta) = V_0 \sum_{s=1}^{N} \sum_{m=-\infty}^{+\infty} i^m e^{-im\varphi_t} c_{sm} TW_m(\xi-\chi_s, \zeta-\eta_s, \chi_s)$$

CWA: Theoretical Method

Boundary Conditions

- E polarization $$\left[V_{\mathrm{t}}(\xi,\zeta)+V_{\mathrm{s}}(\xi,\zeta)+V_{\mathrm{sr}}(\xi,\zeta)\right]_{\rho_{\mathrm{s}}=\alpha_{\mathrm{s}}}=0$$
- H polarization $$\left[\frac{\partial V_{\mathrm{t}}(\xi,\zeta)}{\partial\rho_{\mathrm{s}}}+\frac{\partial V_{\mathrm{s}}(\xi,\zeta)}{\partial\rho_{\mathrm{s}}}+\frac{\partial V_{\mathrm{sr}}(\xi,\zeta)}{\partial\rho_{\mathrm{s}}}\right]_{\rho_{\mathrm{s}}=\alpha_{\mathrm{s}}}=0$$

$$\sum_{\mathrm{q}=1}^{N}\sum_{\mathrm{m}=-\infty}^{+\infty}A_{\mathrm{pm}}^{\mathrm{qs}}c_{\mathrm{qm}}-B_{\mathrm{p}}^{\mathrm{s}}=0$$

$$\mathrm{s}=1,\ldots,N,\ \mathrm{p}=0,\pm1,\pm2,\ldots,$$

$$B_{\mathrm{p}}^{\mathrm{s}}=-T_{01}(n_{\parallel}^{\mathrm{i}})G_{\mathrm{p}}(n_1\alpha_s)e^{in_1(n_{\perp}^{\mathrm{t}}\chi_{\mathrm{s}}+n_{\parallel}^{\mathrm{t}}\eta_{\mathrm{s}})}e^{-i\mathrm{p}\varphi_{\mathrm{t}}}$$

$$A_{\mathrm{pm}}^{\mathrm{qs}}=i^{\mathrm{m-p}}e^{-i\mathrm{m}\varphi_{\mathrm{t}}}G_{\mathrm{p}}(n_1\alpha_s)\left\{CW_{\mathrm{m-p}}(n_1\xi_{\mathrm{qs}},n_1\zeta_{\mathrm{qs}})(1-\delta_{\mathrm{qs}})+\frac{\delta_{\mathrm{qs}}\delta_{\mathrm{mp}}}{G_{\mathrm{p}}(n_1\alpha_s)}+\right.$$
$$\left.+RW_{\mathrm{m+p}}\left[-n_1(\chi_{\mathrm{q}}+\chi_{\mathrm{s}}),n_1(\eta_{\mathrm{s}}-\eta_{\mathrm{q}})\right]\right\}$$

where $G_{\mathrm{p}}(\cdot)=J_{\mathrm{p}}(\cdot)/H_{\mathrm{p}}^{(1)}(\cdot)$ for E pol. and $G_{\mathrm{p}}(\cdot)=J'_{\mathrm{p}}(\cdot)/H'^{(1)}_{\mathrm{p}}(\cdot)$ for H pol.

CWA: Theoretical Method

Monochromatic plane-wave scattering problem by N dielectric circular cylinders buried in a dielectric half-space

(Radio Science, 40, 2005, RS6S18)

Field Inside the q-th Cylinder

$$V_{cq}(\xi_q, \zeta_q) = V_0 \sum_{m=-\infty}^{+\infty} i^m e^{-im\varphi} d_{qm} J_m(n_{cq}\rho_q) e^{im\theta_q}$$

unknown coefficients

Boundary Conditions

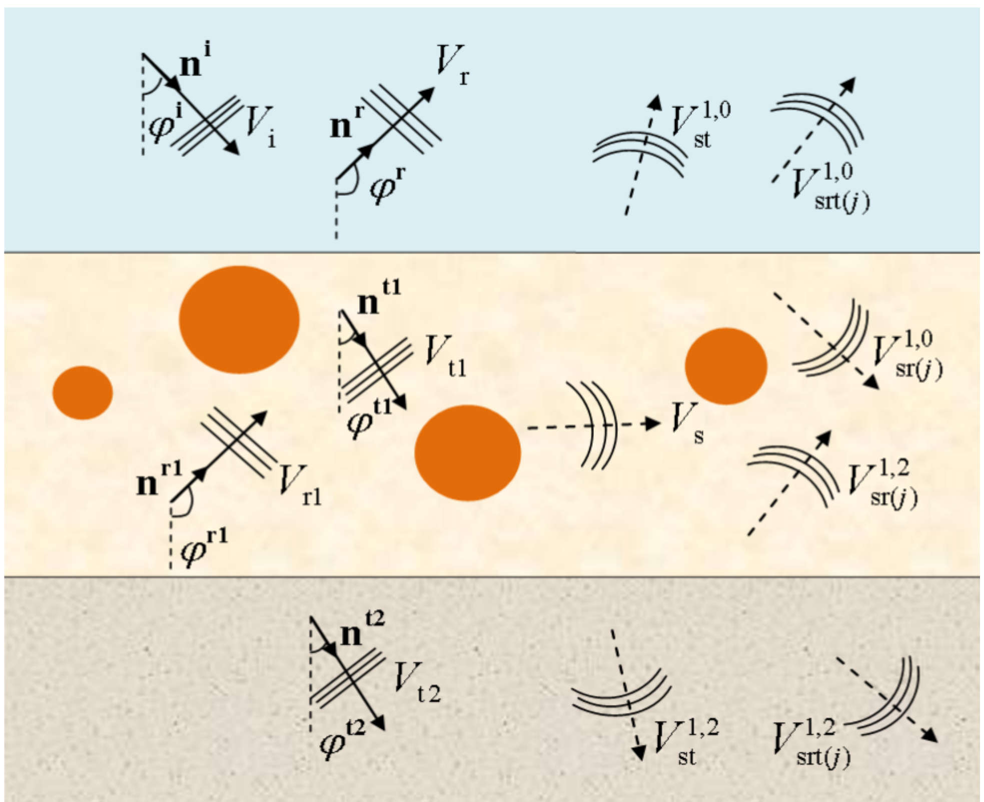
$$\begin{cases} V_t + V_s + V_{sr}\big|_{\rho_q=\alpha_q} = V_{cq}\big|_{\rho_q=\alpha_q} \\ \dfrac{\partial}{\partial\rho_q}(V_t+V_s+V_{sr})\Big|_{\rho_q=\alpha_q} = t_q \dfrac{\partial V_{cq}}{\partial\rho_q}\Big|_{\rho_q=\alpha_q} \end{cases}$$

where $t_q = 1$ for E pol. and $t_q = (n_1/n_{cq})^2$ for H pol.

CWA: Theoretical Method

Monochromatic plane-wave scattering problem by N perfectly-conducting and dielectric circular cylinders buried <u>in a finite-thickness slab</u>

(IEEE Trans. Antennas and Propagation, 57(4), 2009, pp. 1208-1217,
Journal of Optical Society of America A, 27(4), 2010, 687-695)



- With the introduction of more interfaces between different media, the definition of

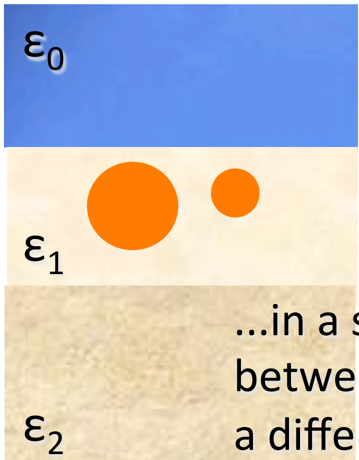
  generalized multiple-reflected cylindrical functions

  and

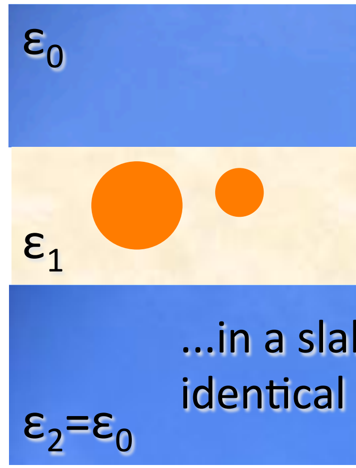
  multiple-reflected-transmitted cylindrical functions

  relevant to multiple reflection phenomena is needed.

CWA: Theoretical Method

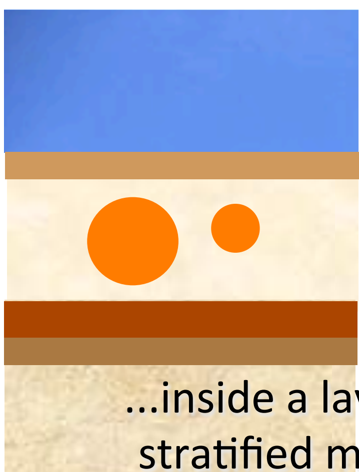


...in a soil slab, between air and a different soil

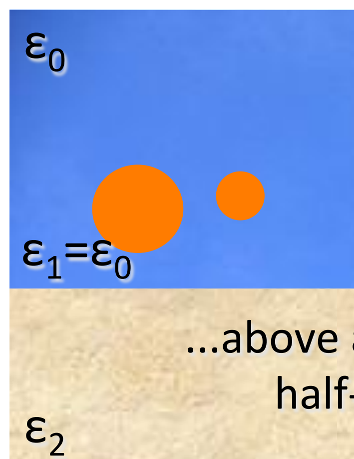


...in a slab, among two identical half-spaces

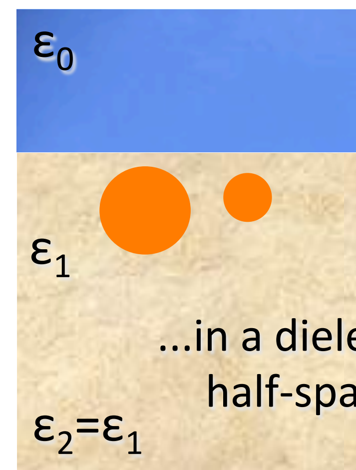
...in a grounded slab



...inside a layer of a stratified medium



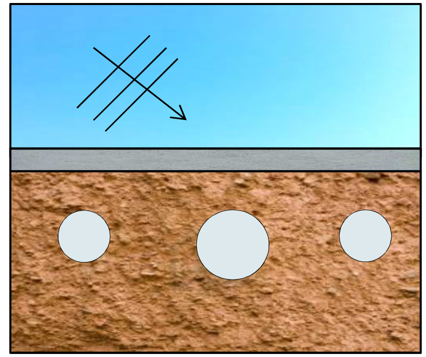
...above a dielectric half-space

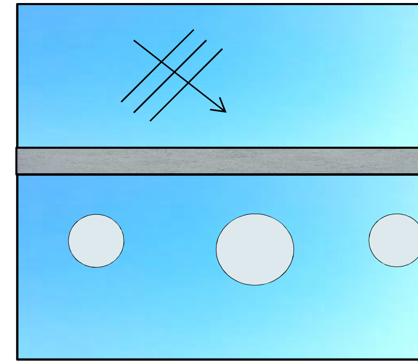


...in a dielectric half-space

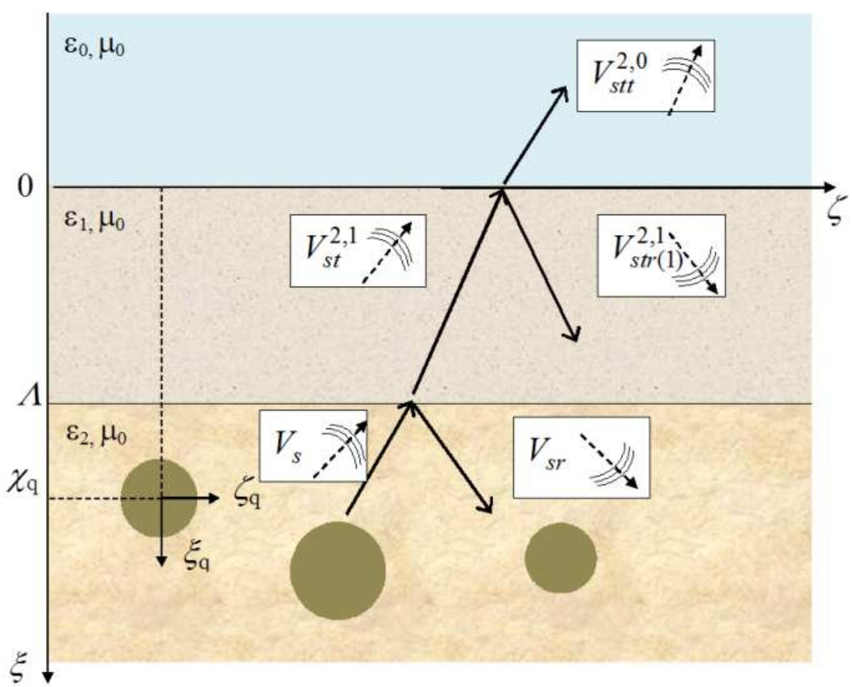
CWA: Theoretical Method

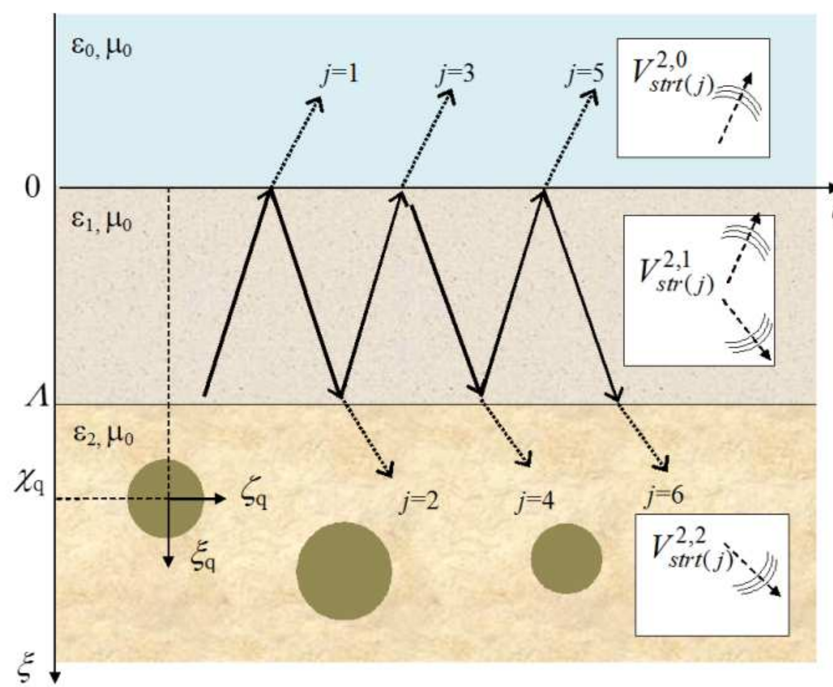
- Targets buried in a non-homogeneous soil or below asphalt layer
- Through wall scattering





(JOSA A, vol. 30(8), 2013, pp. 1632-1639)





CWA: Theoretical Method

Pulsed plane-wave scattering problem by N perfectly-conducting and dielectric circular cylinders buried in a dielectric half-space

(IEEE Geoscience Remote Sensing Letters, 4(4), 2007, pp. 611-615)

- Sampling of the incident-field spectrum and of the spectra of the various field terms
- Solution for any sample in the spectral domain by using the CWA
- Time-domain solution by means of the inverse transform

Current Line scattering problem by N perfectly-conducting and dielectric circular cylinders buried in a dielectric half-space

(Progress in Electromagnetics Research, 80, 2008, pp. 179-196)

$$V_i(\xi,\zeta) = -V_0 H_0^{(1)}\left[n_1\sqrt{(\xi-\chi_L)^2+(\zeta-\eta_L)^2}\right]$$

CWA: Theoretical Method

Arbitrary 2D distribution of the field

Near Surface Geophysics, 13(3), 2015, pp. 219-225

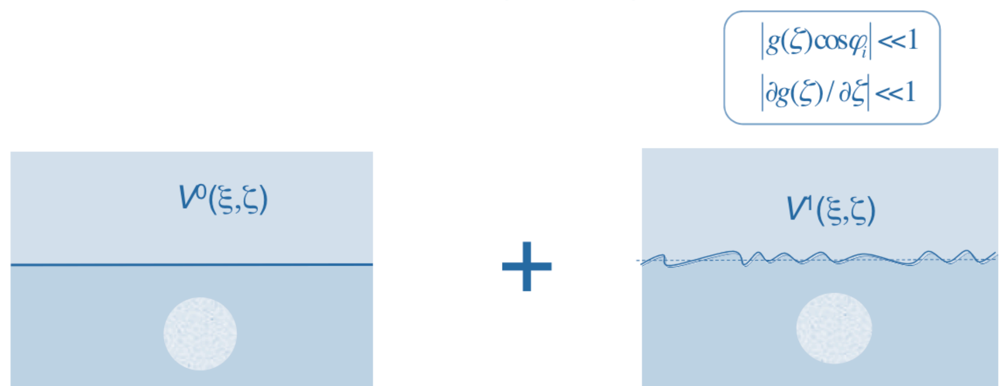
Expansion of the incident field in plane waves. CWA applied to each plane wave.

Rough surface between air and soil

Near Surface Geophysics, 11(2), 2013, pp. 177-183

Small Perturbation Method combined with the CWA

Superposition of a flat boundary + slightly rough deviations $\xi = g(\zeta)$



CWA zero-order solution
unperturbed fields

CWA first-order solution
perturbed fields

CWA: Numerical solution

CWA computational issues

Accuracy

Depends on the truncation of the involved series (cylindrical expansions)

Memory requirements

Low.

Execution time

Increases with the number of cylinders, their size and the permittivity of the involved materials.

CWA: Numerical solution

Series truncation to a finite number of elements

$$\sum_{m=-\infty}^{+\infty} \Rightarrow \sum_{m=-M_t}^{+M_t}$$

- Convergence properties of the method
- Truncation criterion

$$M_t = 3 n_1 \alpha_{MAX}$$

Numerical evaluation of spectral integrals

- Infinite extension of the integration domain
- Highly oscillating behavior of the integrand
- Considerable variability of the $F_m(\xi, n_{||})$ function

$$TW_m(\xi,\zeta,\chi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} T_{10}(n_{\|}) F_m(-n_1\chi, n_{\|}) e^{i n_1 n_{\|} \zeta} e^{-i(\xi+\chi)\sqrt{1-(n_1 n_{\|})^2}} dn_{\|}$$

- Short computational time is desirable for inverse scattering applications

CWA: Numerical solution

Integration of the evanescent spectrum

- Smaller ζ values: Laguerre-Gauss quadrature formula
- Larger ζ values: decomposition of the integration interval in subintervals of suitable length, Gauss-Legendre quadrature formula in each subinterval, ε-algorithm (convergence acceleration)

Integration of the homogeneous spectrum

- Discrimination between partially and totally reflected waves
- Development of an adaptive integration technique

✓ Calculus of the local frequency oscillation rate f

✓ Decomposition of the integration interval in subintervals in which f behaves monotonically; further decomposition of each subinterval, by using an adaptive procedure for the evaluation of the effective oscillation period

✓ Gauss-Legendre quadrature formula in each sub-subinterval

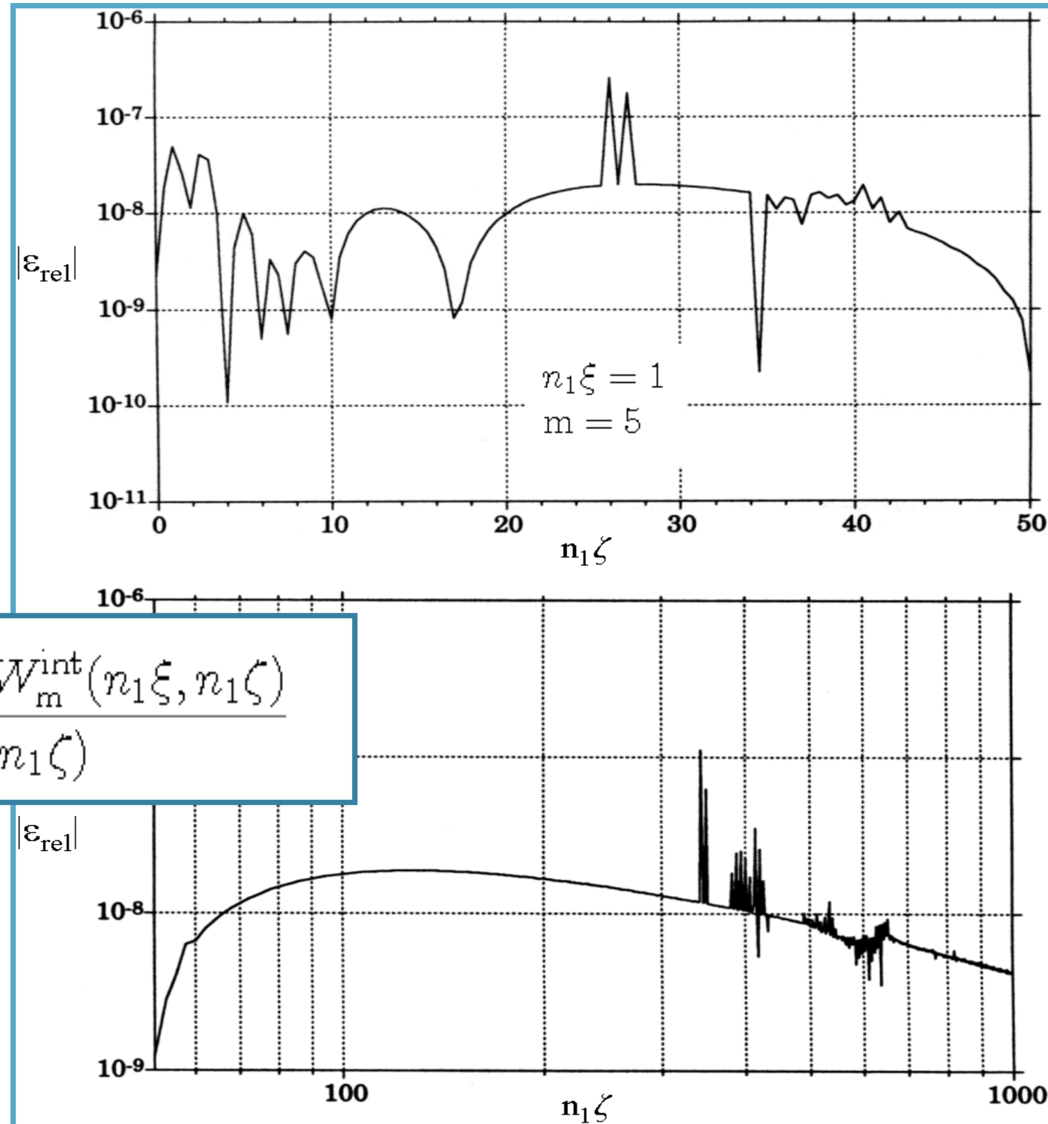
CWA: Numerical solution

- If T_{10}=K and χ= - ξ:

  $$TW(\xi,\zeta,-\xi) = KCW(n_1\xi, n_1\zeta)$$

quantitative comparison between the results of the integration algorithm and an accurate evaluation of CW carried out by means of 20th order polynomial approximation of Bessel functions

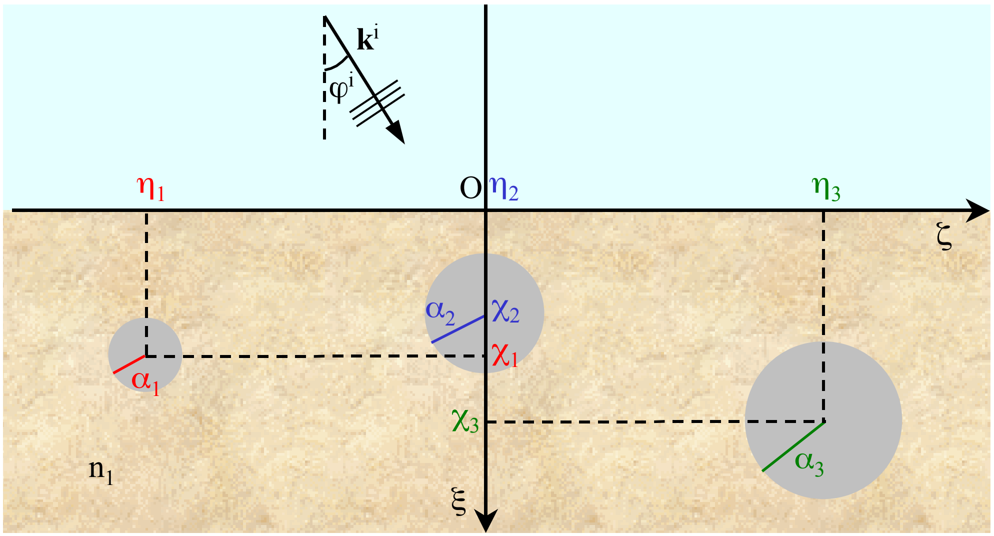
$$\varepsilon_{\rm rel}(n_1\xi, n_1\zeta) = \frac{CW_{\rm m}^{\rm def}(n_1\xi, n_1\zeta) - CW_{\rm m}^{\rm int}(n_1\xi, n_1\zeta)}{CW_{\rm m}^{\rm def}(n_1\xi, n_1\zeta)}$$



- This case corresponds to a flat surface with reflection properties independent of the incident angle

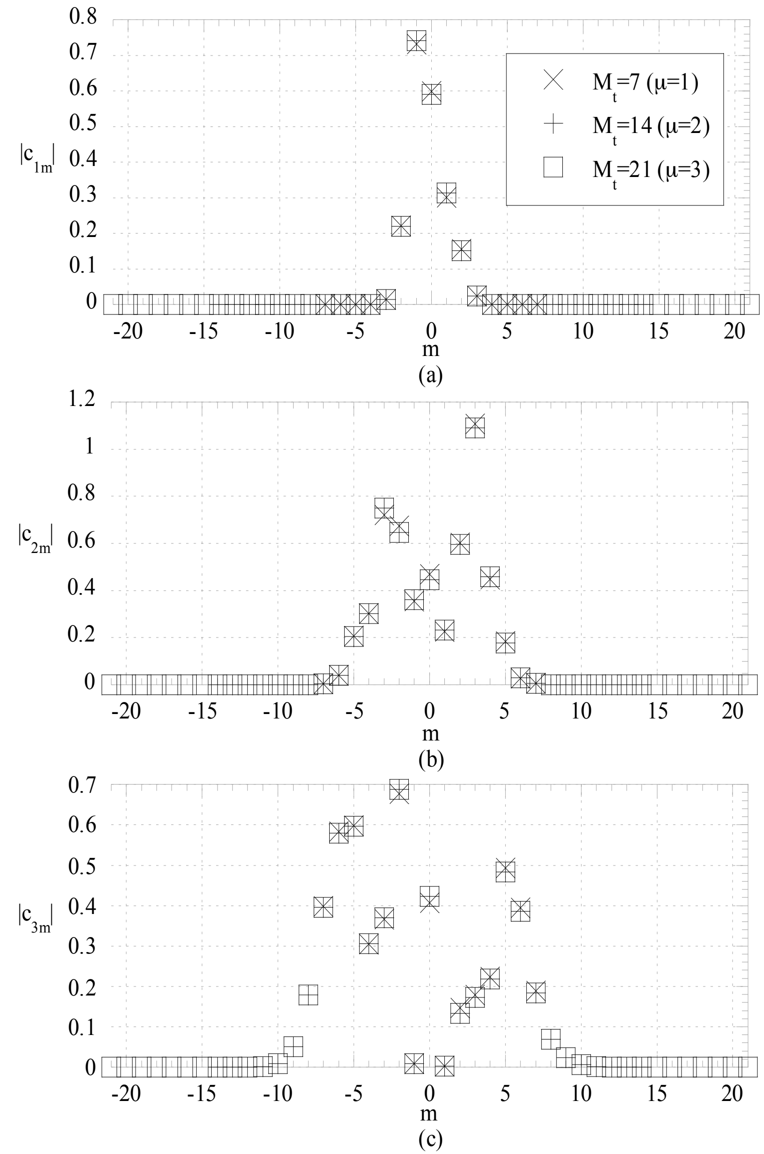
CWA: Numerical solution

Convergence properties of the CWA



$\alpha_1 = 1$, $\alpha_2 = 3$, $\alpha_3 = 5$, $\chi_1 = 10$, $\chi_2 = 5$, $\chi_3 = 15$, $\eta_1 = -10$, $\eta_2 = 0$, $\eta_3 = 10$, $n_1 = 1.4$, $\varphi_i = 45°$, $N = 3$

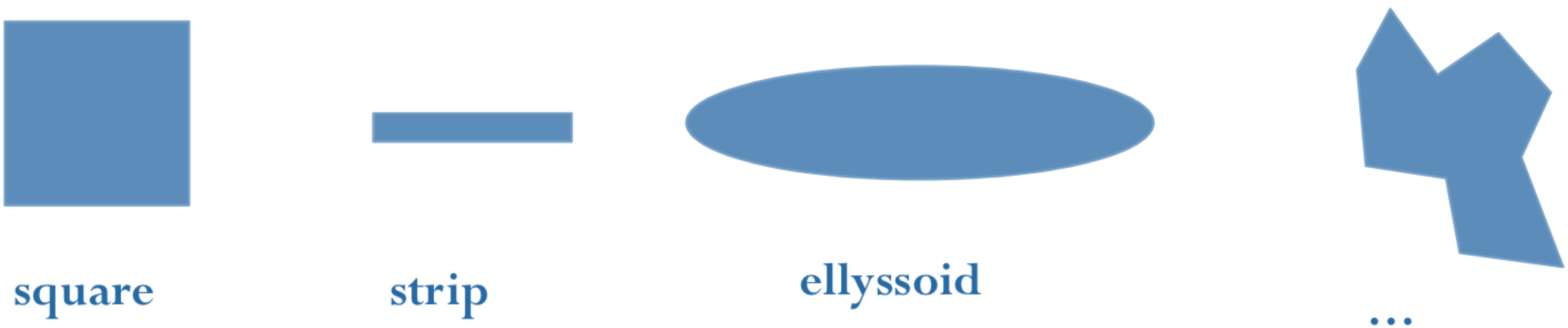
$$M_t = 3 n_1 \alpha_{MAX}$$



CWA: Numerical results

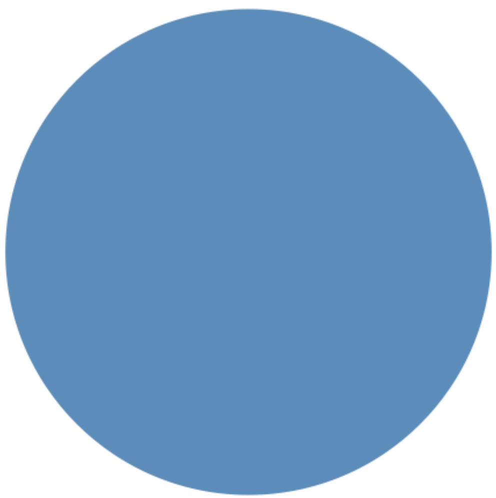
CWA is conceived for the simulation of cylindrical targets with circular section.

But also targets with arbitrary shape can be simulated!!

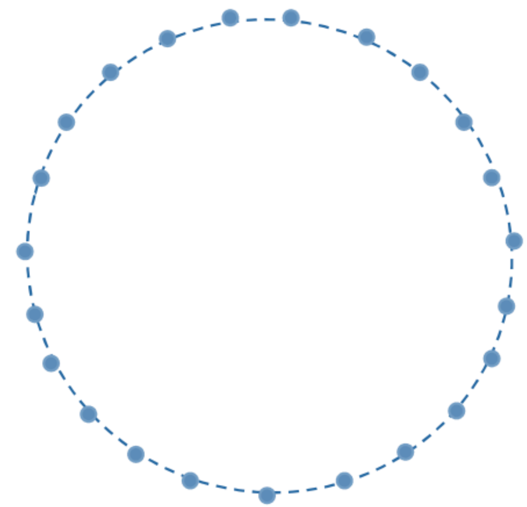


CWA: Numerical results

The arbitrary shape of a PEC target can be approximated with a suitable set of PEC small circular-section cylinders along ist border.



original shape



approximation with a wire-grid model

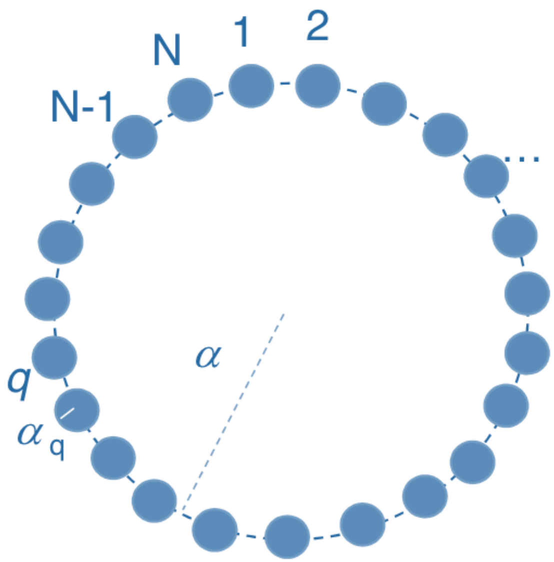
CWA: Numerical results

The total area of the approximating small cylinders has to be equal to the area of the simulated target.

Same Area Rule

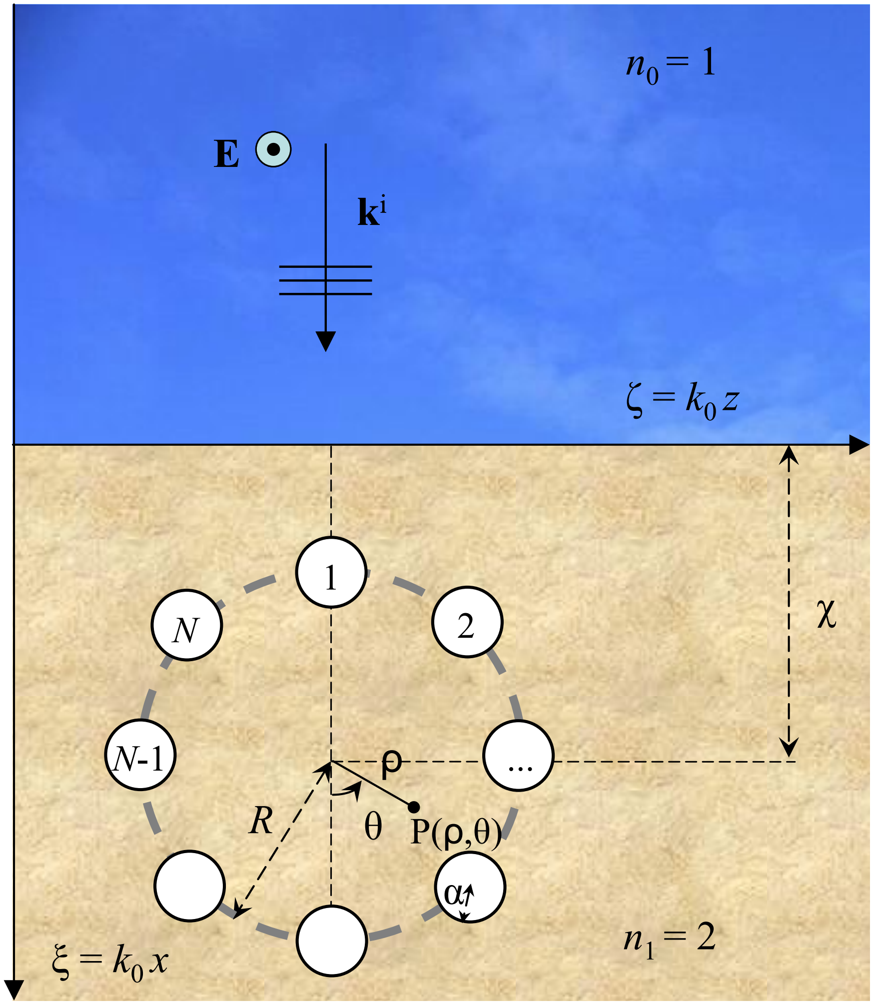
$$2\pi\alpha = N 2\pi\alpha_q$$

$$\Rightarrow \quad \alpha_q = \frac{\alpha}{N}$$



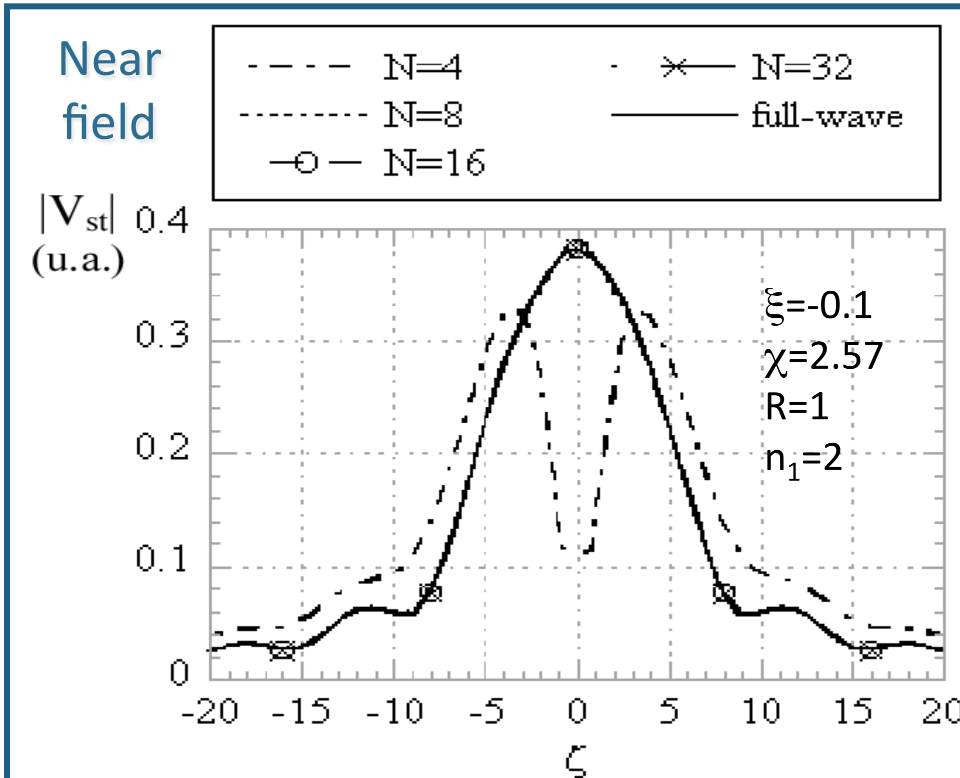
Once N is fixed, the same area rule allows to calculate the size of the small cylinders. Such rule is not the optimum, but gives good results. Configurations of cylinders of different size may also be used.

CWA: Numerical results

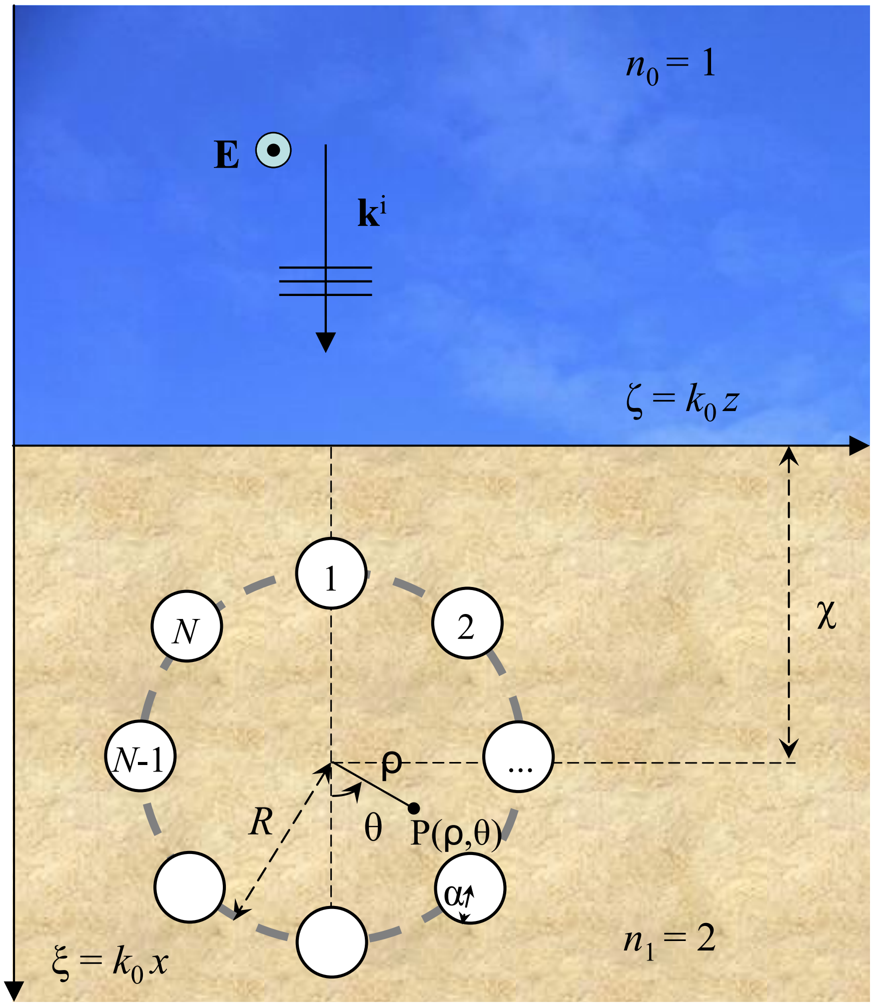


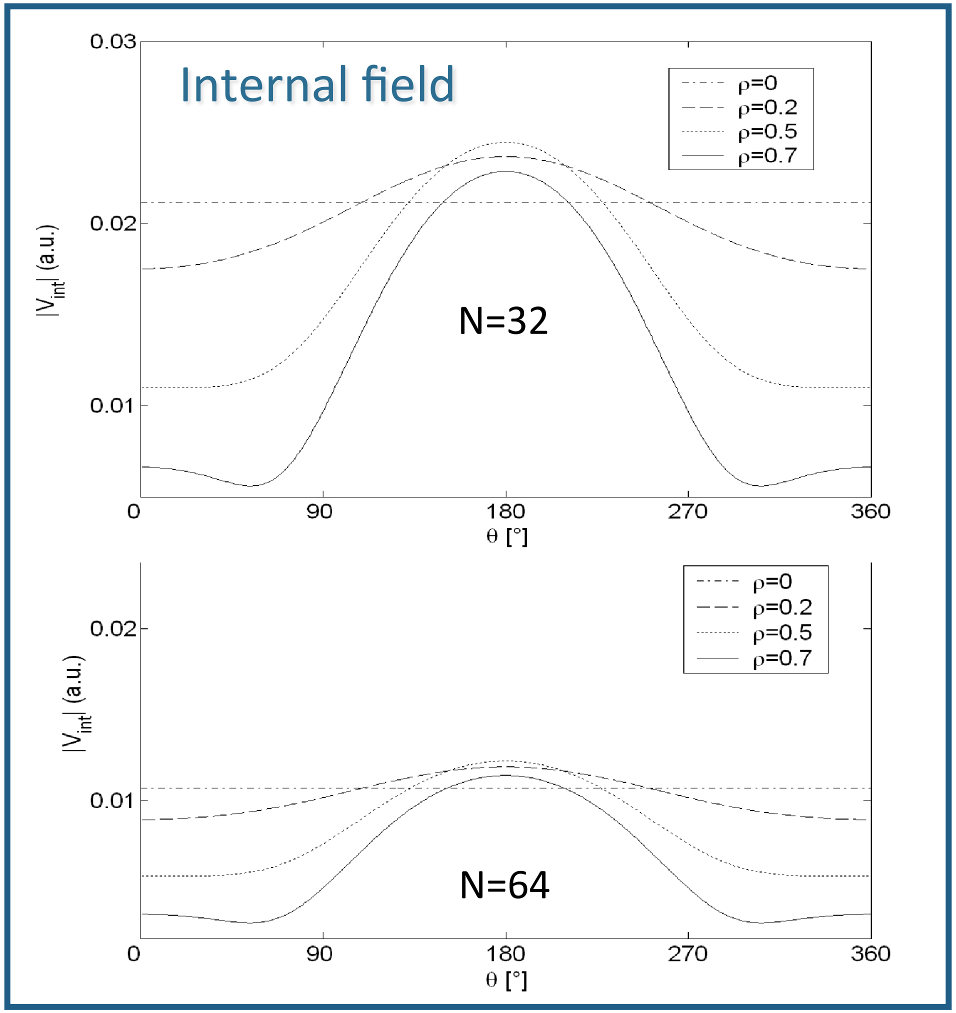
Same-Area Rule

2πR = N 2πα ⇒ α = R/N



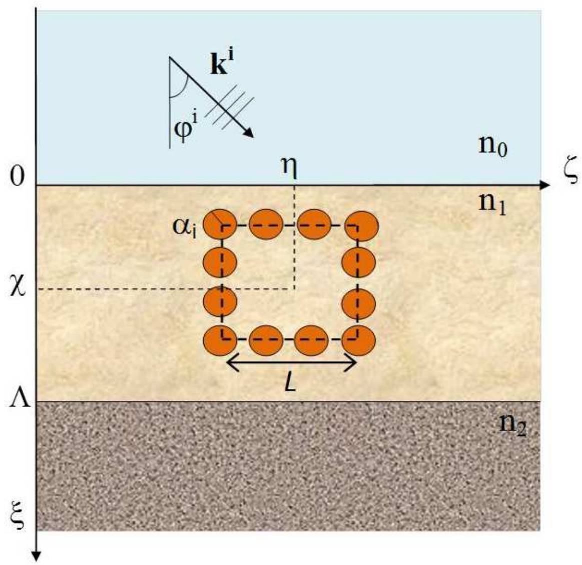
CWA: Numerical results



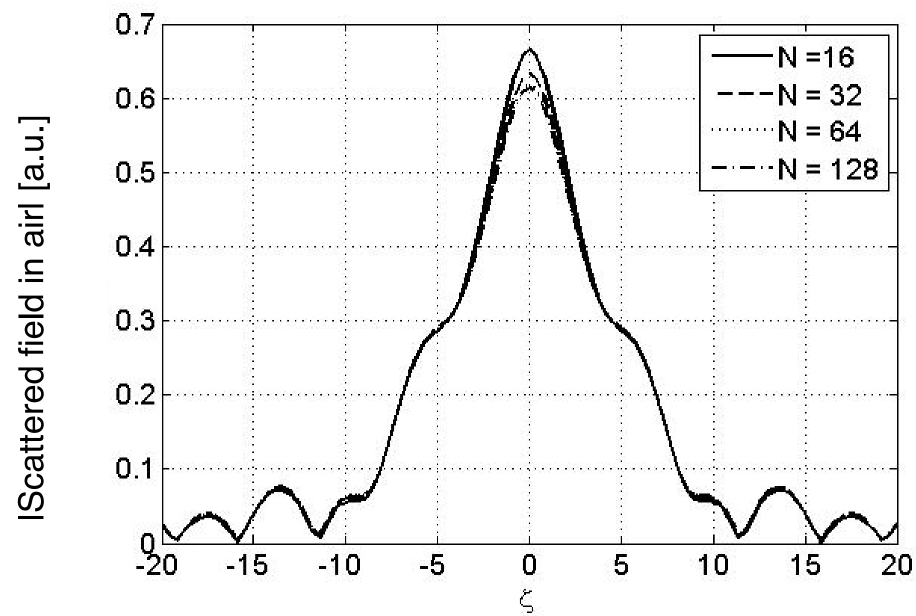


CWA: Numerical results

Wire-grid simulation of a square metallic border



Near field: $\xi = -0.1$



$L = 2$
$\chi = 2.6\pi$
$\Lambda = 50$
$\varphi_i = 0$
TM polarization

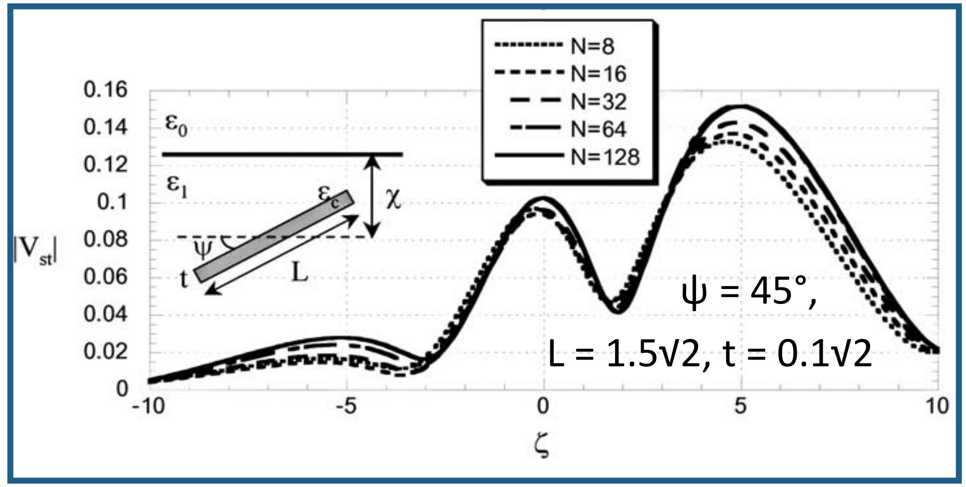
$n_1 = \sqrt{2}$
$n_2 = \sqrt{1.5}$

CWA: Numerical results

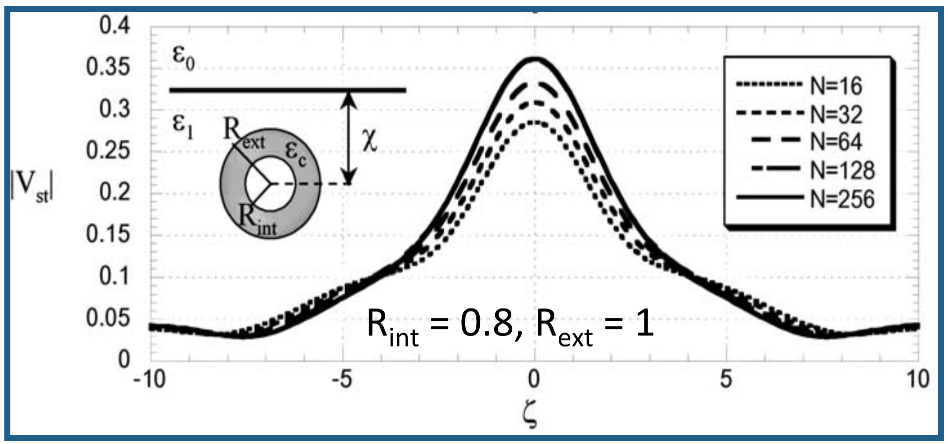
Same-Volume Rule

Good results are obtained when the total volume (for unit length) of the N modeling scatterers is equal to the volume of the simulated object

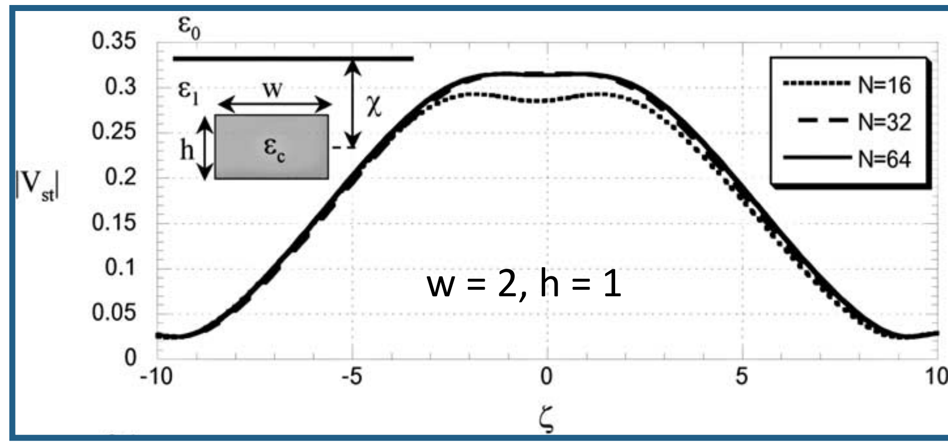
Tilted Slab



Circular Shell



Rectangular-Section Scatterer



$n_1 = 2$, $n_c = 5$, $\chi = 2.5$, $\xi = 1$, $\phi^i = 0$, E pol.

CWA: Numerical results

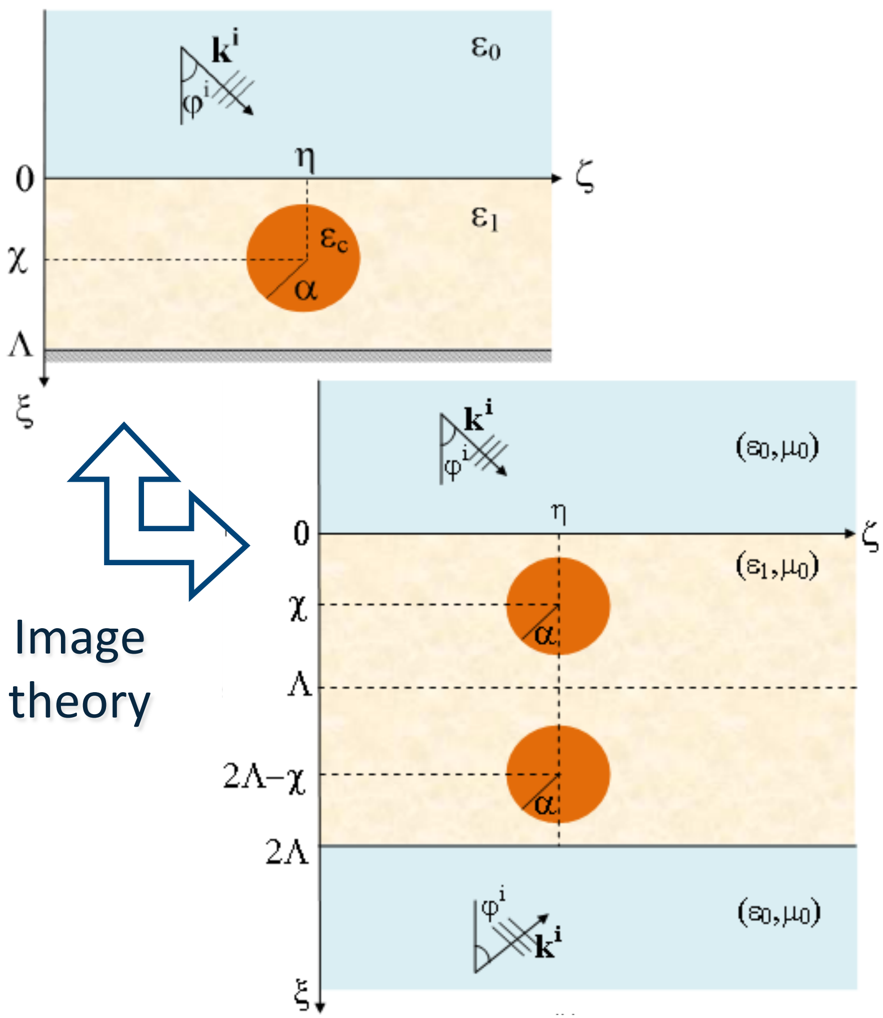
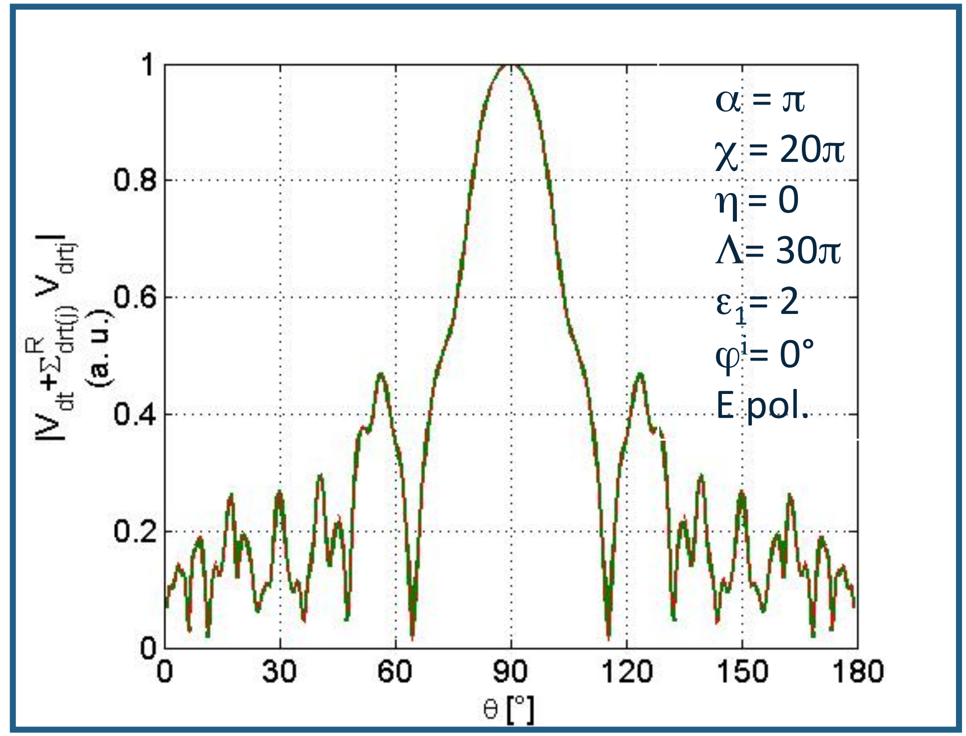


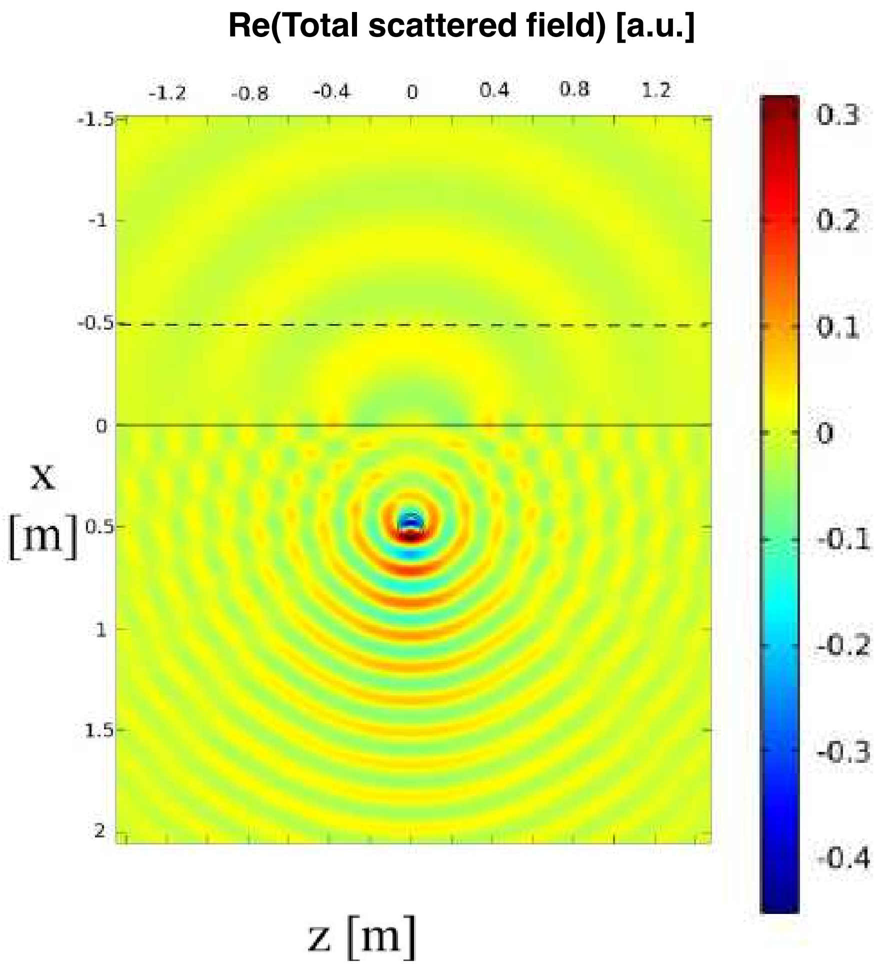
Image theory



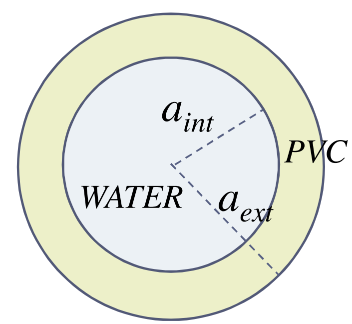
PEC cylinder in a grounded slab

Equivalent (for $\xi < \Lambda$) geometry

CWA: Numerical results



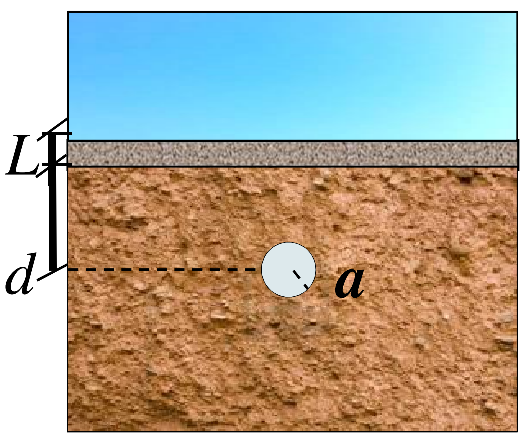
$\varepsilon'_1 = 10$	$\sigma_1 = 0.01$ S/m
$\varepsilon'_W = 81$	$\sigma_W = 1$ S/m
$\varepsilon'_{PVC} = 3.7$	$\sigma_1 = 0$
$a_{int} = 5$ cm	$a_{ext} = 10$ cm
$h = 50$ cm	$f = 600$ MHz
TM pol.	$h_L = -1, d_L = 0$

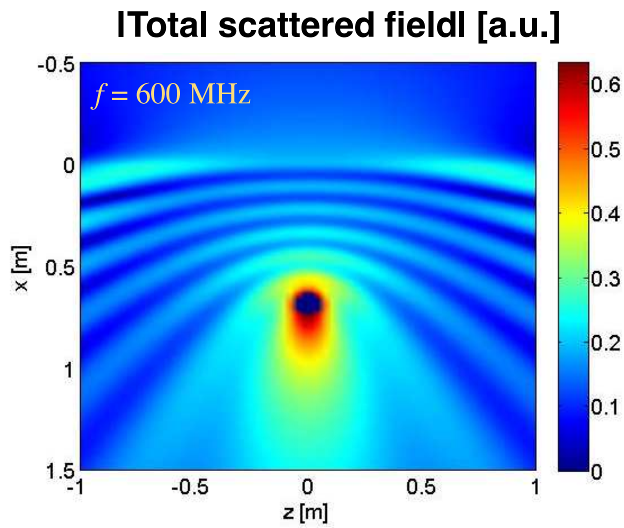


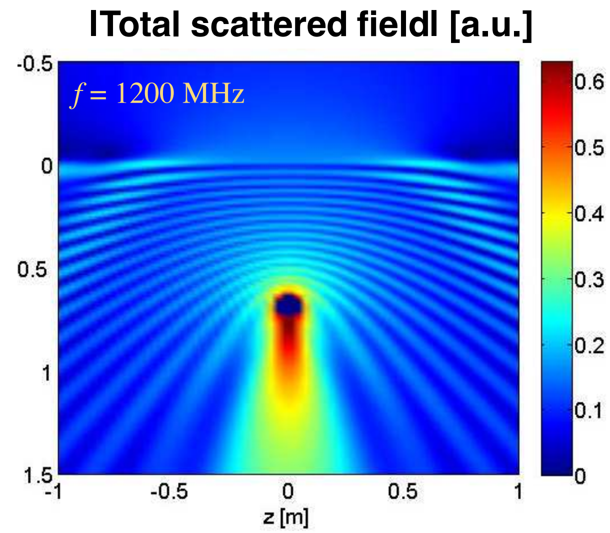
CWA: Numerical results

- Air-dielectric-dielectric background
- Buried metallic cable

$a = 5.5$ cm	$\varepsilon_{r1} = 3.5$
$d = 60$ cm	$\varepsilon_{r2} = 5$
$L = 10$ cm	TM pol.
$\varphi_i = 0$	



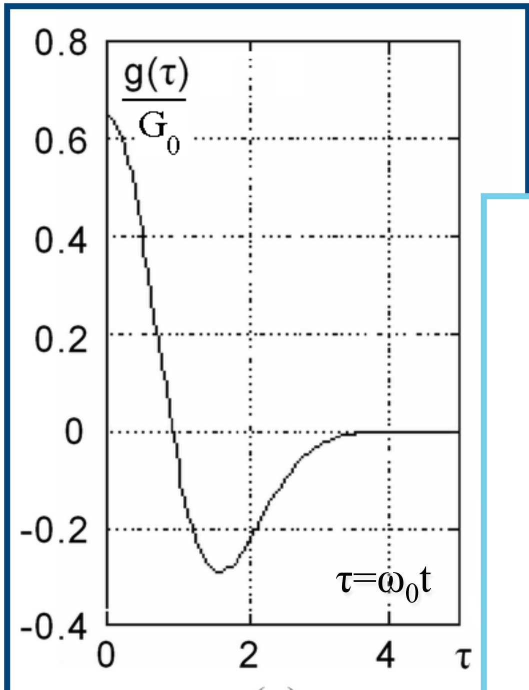




CWA: Numerical results

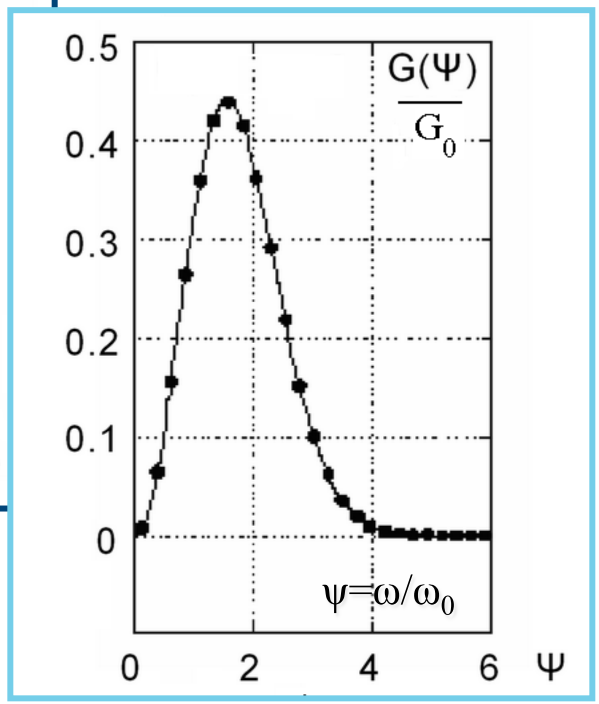
- Time-shape of the incident pulsed plane-wave:

$$g\left(\tau - \mathbf{n}^{\mathrm{i}}\cdot\boldsymbol{\rho}\right) = g_0 e^{-\frac{\left(\tau-\mathbf{n}^{\mathrm{i}}\cdot\boldsymbol{\rho}\right)}{2}}\left[e^{\frac{1}{2}}\cos\left(\tau-\mathbf{n}^{\mathrm{i}}\cdot\boldsymbol{\rho}\right)-1\right]$$



- The spectrum and its samples:

$$H(\psi,\boldsymbol{\rho}) = G(\psi)\, e^{i\psi(\xi\cos\varphi_{\mathrm{i}}+\zeta\sin\varphi_{\mathrm{i}})} =$$
$$= g_0\frac{\sqrt{2\pi}}{\omega_0}e^{-\frac{\psi^2}{2}}\left[\cosh(\psi)-1\right]e^{i\psi\mathbf{n}^{\mathrm{i}}\cdot\boldsymbol{\rho}}$$



✓ Waveform consistent with many current ultrawide-band short-pulsed sources

✓ This pulse can be considered vanishing for τ>4 and its spectrum does not contain frequencies ψ>6

✓ With an appropriate choice of ω_0 the reference frequency, related to spatial resolution, can be tuned

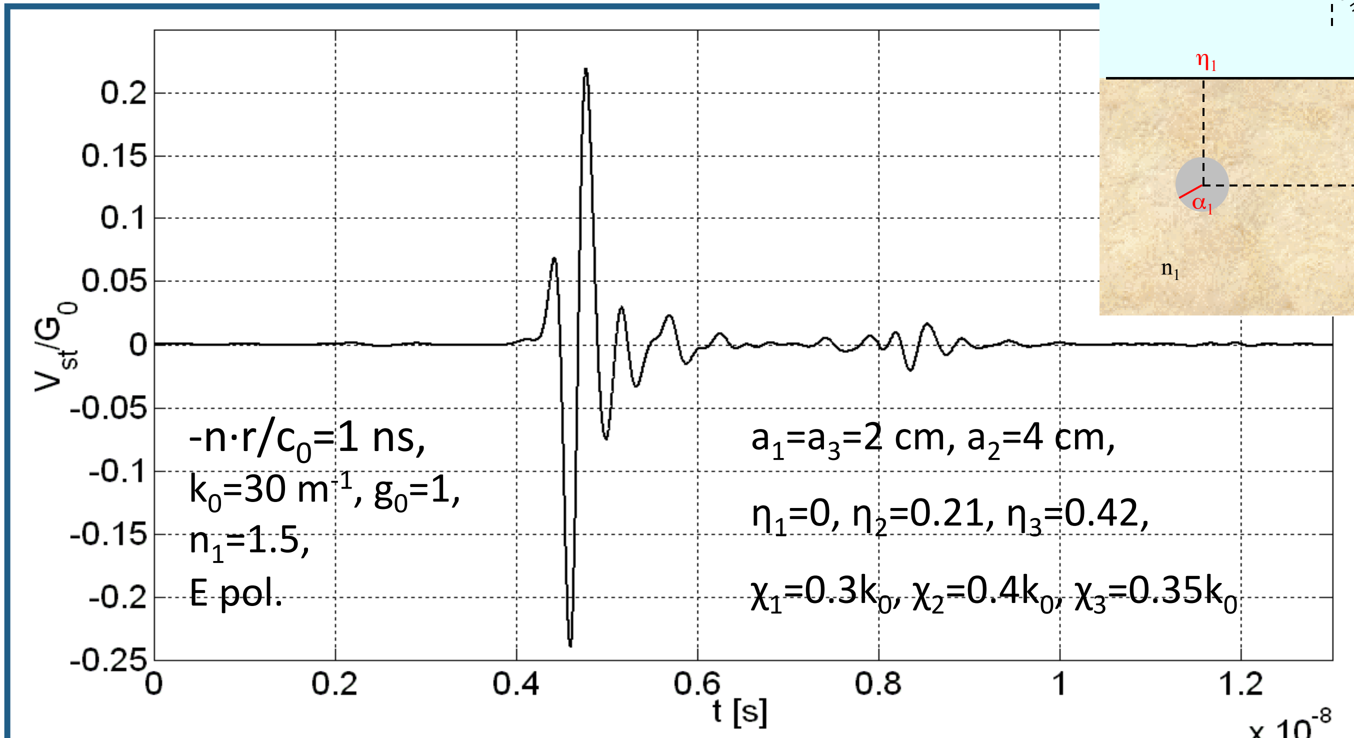
CWA: Numerical results

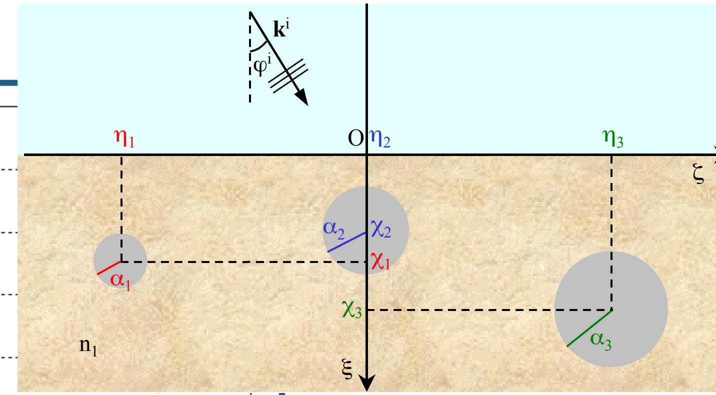
- Time-shape of the scattered-transmitted field:

✓ Diffraction from cylinders

✓ Reverberations between cylinders and air-ground interface

✓ Creeping-wave circumnavigation of cylinders





CWA: Conclusions

- ❑ **CWA: Analytical-numerical method for the solution of the 2D scattering problem by a set of cylinders buried in a dielectric half-space or finite-thickness slab**
  - Adaptive integration techniques for the evaluation of the spectral integrals
  - Source: monochromatic or pulsed plane-wave, current line,
    arbitrary 2D distribution of the field
  - Rough interfaces between different media
  - Scatterers of arbitrary cross-section simulated by a suitable set of circular cylinders
  - Results in near- and far-field zones, E and H polarization, spectral and time domain

- ❑ **Applications:**
  - Characterization of GPR scenarios, discretized in cylindrical cells
  - Reflection and transmission by reinforced concrete
  - Diagnostics of buried utilities (tunnels, conduits, electricity cables, gas or water pipes)
  - Direct solver in iterative algorithms for the solution of inverse scattering problems

- ❑ **Lossy media & Extension of the method to 3D geometries:**
  - See Nicola's presentation!

Thank you!

Join COST Action TU1208!

www.GPRadar.eu lara.pajewski@uniroma1.it

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