Reconfigurable stepped-frequency GPR prototype for civil-engineering and archaeological prospection, developed at the National Research Council of Italy. Examples of application and case studies.

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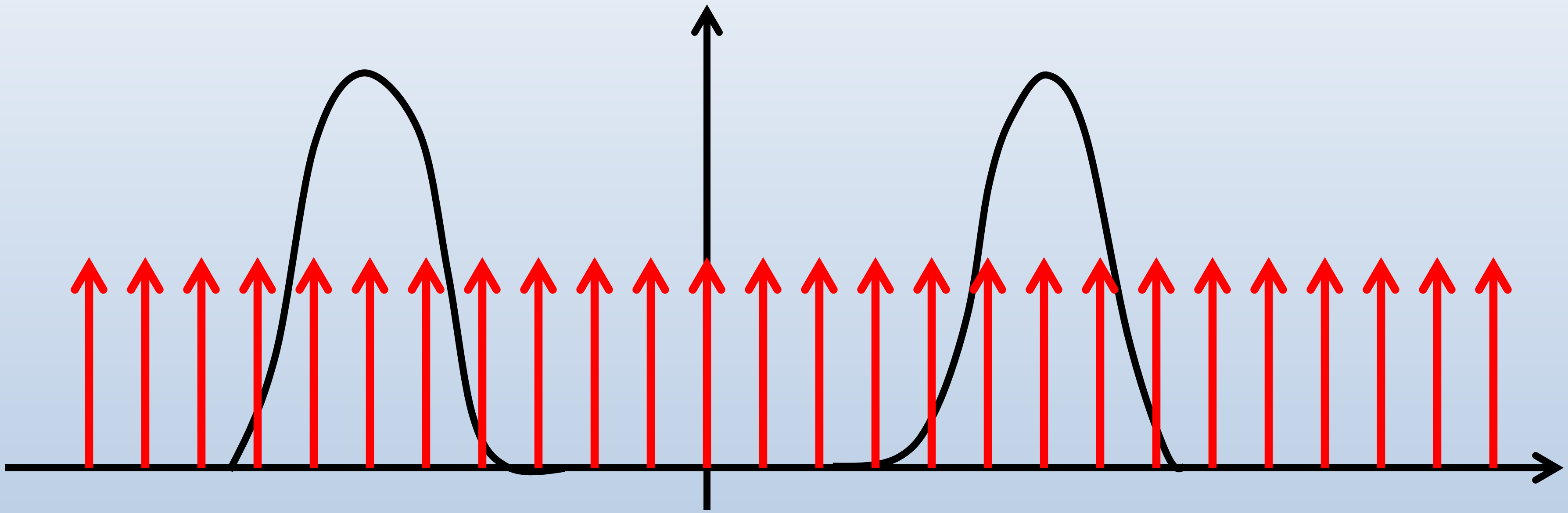
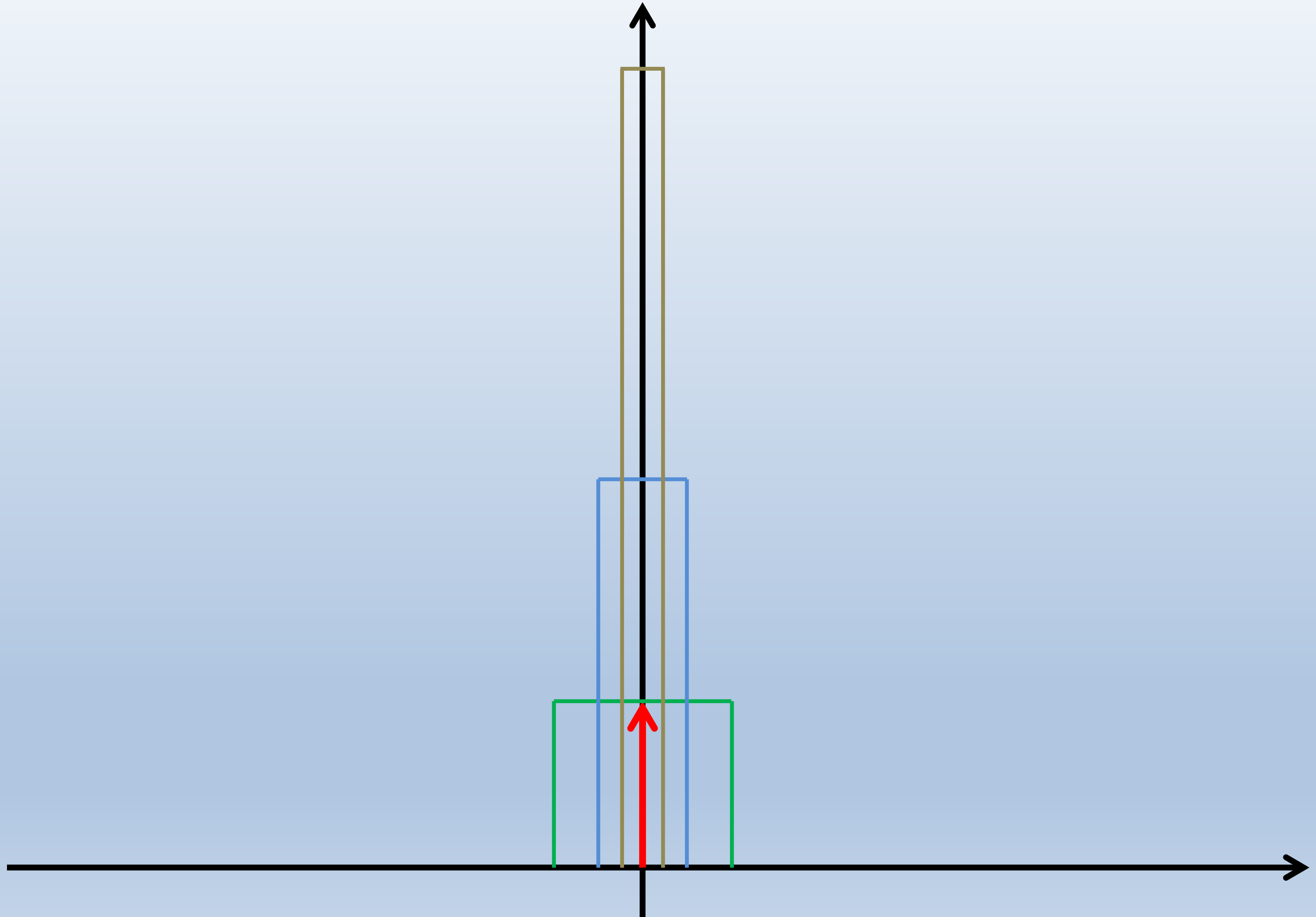
Stepped frequency: underlying concepts



The delta function



Reminds

$$S(f) = \int_{-\infty}^{+\infty} s(t)\exp(-j2\pi ft)\,dt$$

$$S(-f) = \int_{-\infty}^{+\infty} s(t)\exp(j2\pi ft)\,dt = S^{*}(f)$$

$$s(t) = \int_{-\infty}^{+\infty} S(f)\exp(j2\pi ft)\,df =$$

$$= 2\,\mathrm{Re}\int_{0}^{+\infty} S(f)\exp(j2\pi ft)\,df$$

Reminds

Convolution

$$f_1 * f_2 = g(t) = \int_{-\infty}^{+\infty} f_1(\tau) f_2(t-\tau)\, d\tau$$

Convolution and Fourier Transform

$$FT(g_1 * g_2) = G_1(f)\, G_2(f)$$

$$FT(g_1 g_2) = G_1 * G_2$$

Reminds

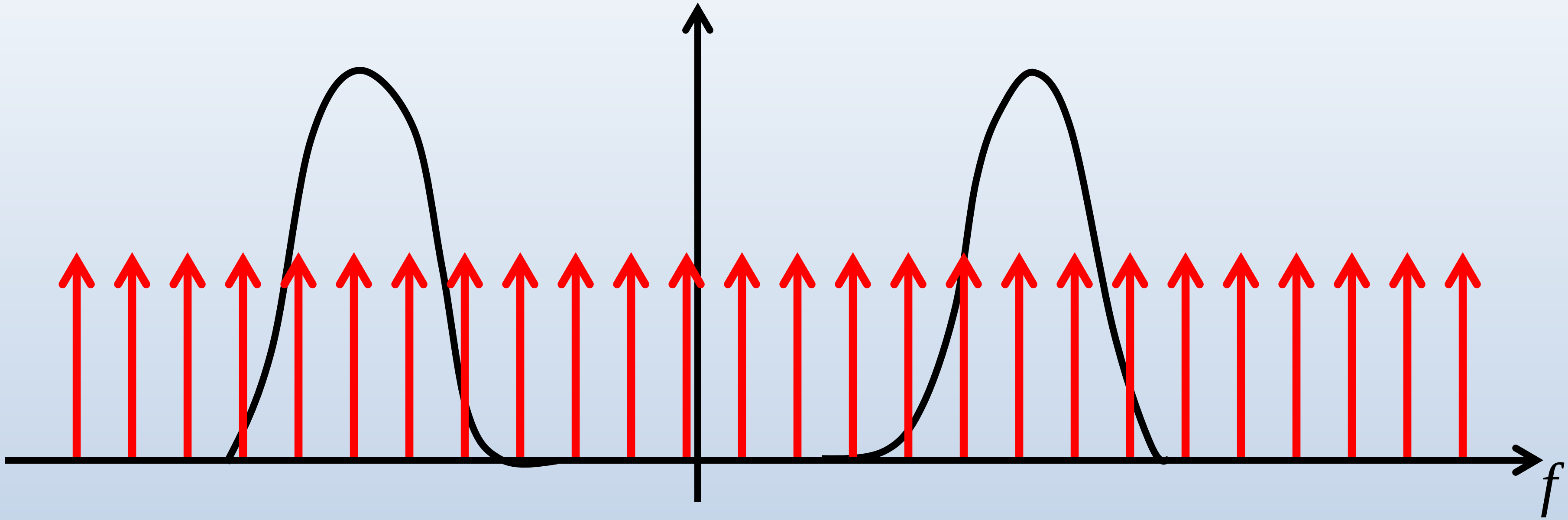
Sampling property of the delta

$$\int_{-\infty}^{+\infty} \delta(\tau) f(\tau) d\tau = f(0)$$

$$\int_{-\infty}^{+\infty} \delta(\tau - \tau_o) f(\tau) d\tau = f(\tau_o)$$

$$f * \delta = \int_{-\infty}^{+\infty} f(\tau) \delta(t - \tau) d\tau = f(t)$$

Ideal sampling of the spectrum



$$2\,\mathrm{Re}\int_0^{+\infty} S(f)\exp(j2\pi ft)\left[\sum_{n=-\infty}^{+\infty}\delta(f-f_o+n\Delta f)\right]df =$$

$$=2\,\mathrm{Re}\left\{\sum_{n=1}^{N} S(f_o+n\Delta f)\exp\big(j2\pi(f_o+n\Delta f)t\big)\right\}=$$

$$2\sum_{n=1}^{N}\Big[\mathrm{Re}\,S(f_o+n\Delta f)\cos\big(2\pi(f_o+n\Delta f)t\big)-\mathrm{Im}\,S(f_o+n\Delta f)\sin\big(2\pi(f_o+n\Delta f)t\big)\Big]=$$

$$2\sum_{n=1}^{N}=\left|S(f_o+n\Delta f)\right|\cos\big[2\pi(f_o+n\Delta f)t+\angle S(f_o+n\Delta f)\big]$$

Expression of the received sinthetic pulse for 2N+1 samples, constant radiated spectrum, constant attenuation and phase of the reflected signals

$$s(t) \approx 2\Delta f \frac{\sin\left(\left(2N+1\right)\pi\Delta f t\right)}{\sin\left(\pi\Delta f t\right)} \cos\left(2\pi f_c t + \theta\right)$$

The synthetic pulse is replicated, but the replicas are in general not equal to each other

Flux diagram

A sequence of sinusoids is transmitted

$$\cos\left(2\pi f_n t\right) = \cos\left(2\pi\left(f_{\min} + n\Delta f\right)t\right)$$

↓

A sequence of sinusoids is retrieved

$$A_n \cos\left(2\pi f_n t + \varphi_n\right) = A_n \cos\left(2\pi\left(f_{\min} + n\Delta f\right)t + \varphi_n\right)$$

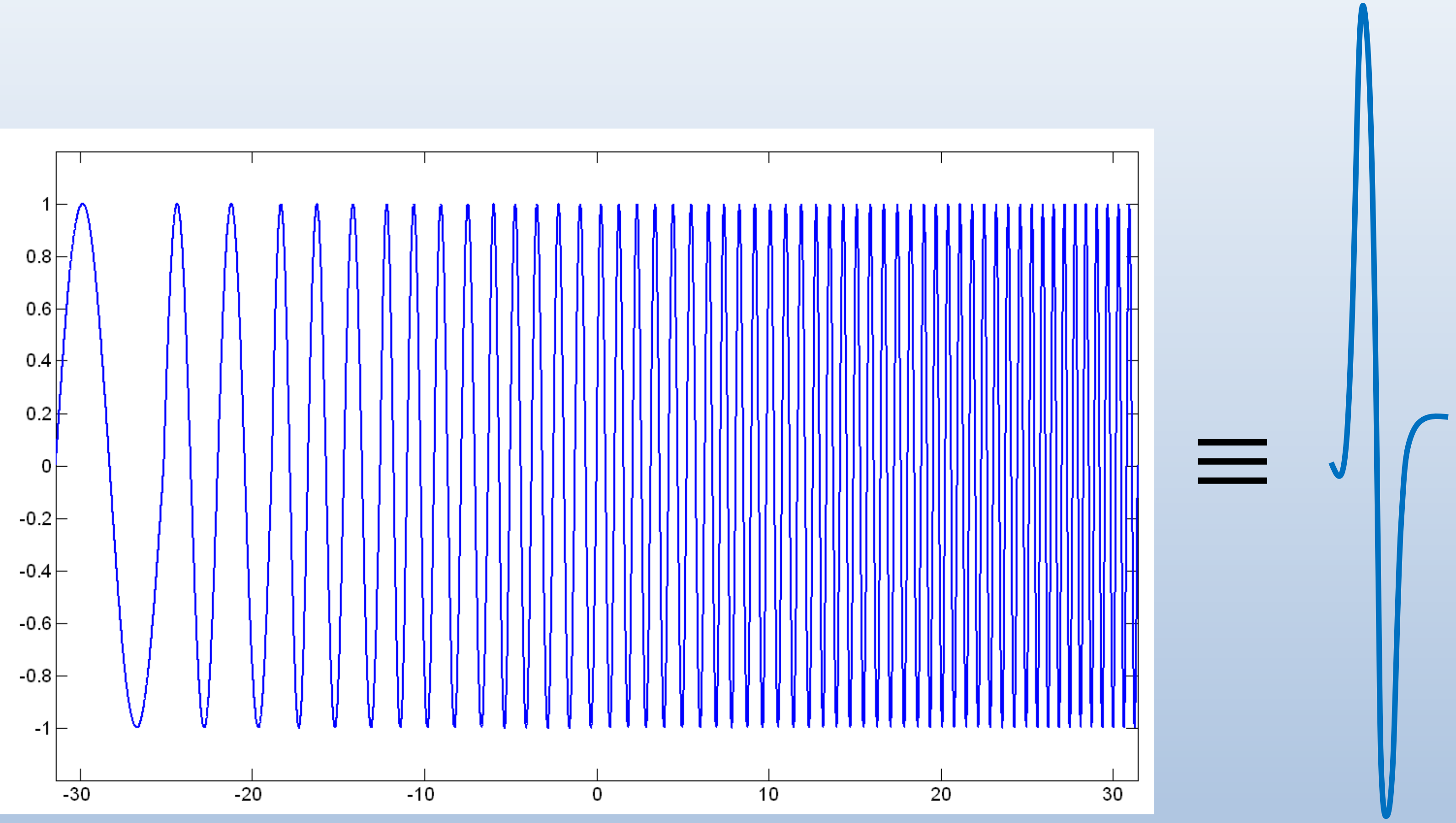
↓

Amplitude and phase A_n and φ_n are extracted

↓

The received harmonics are summed

In “synthesis”…



…but the pulse is replicated with pseudo-replicas at distance 1/Δf …

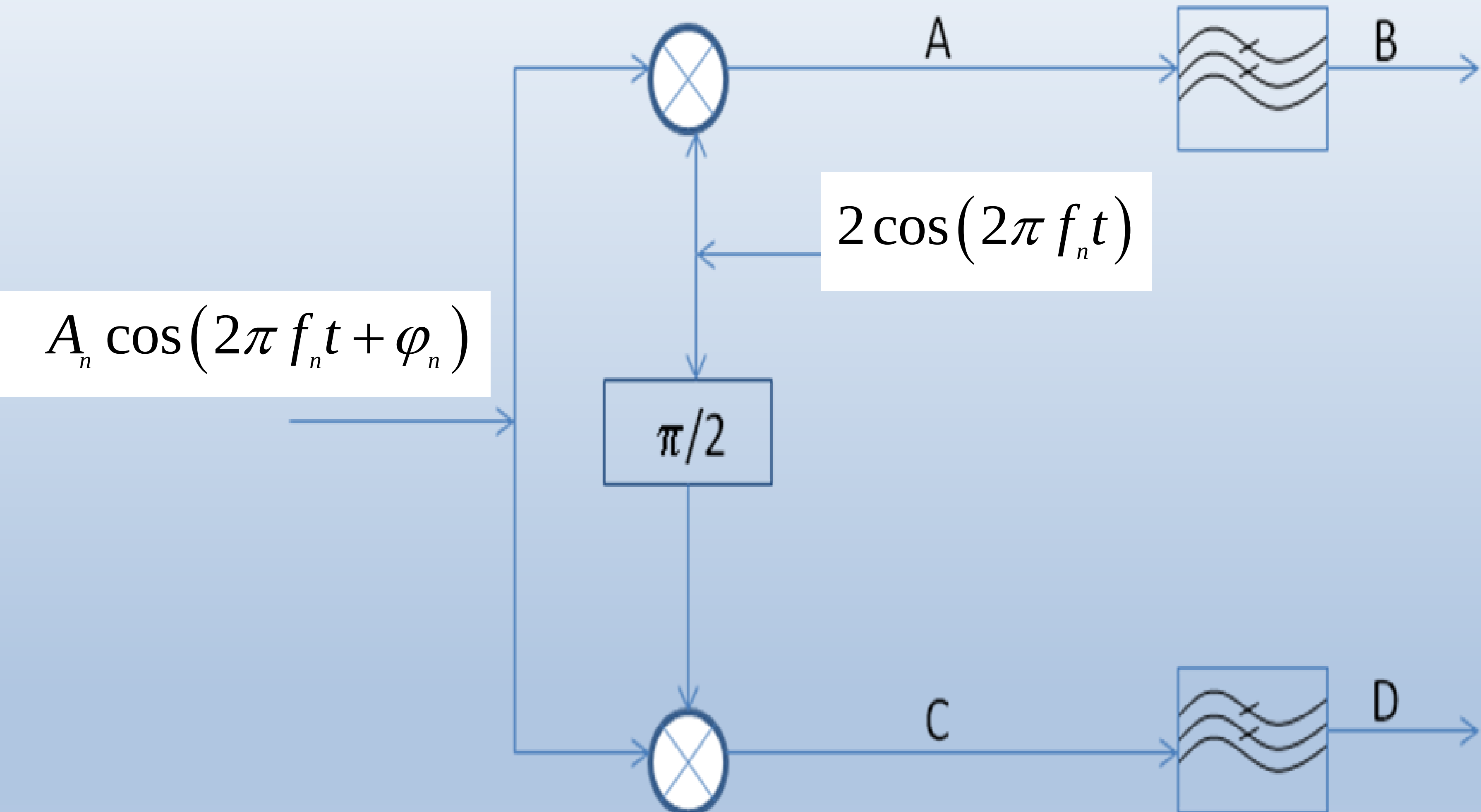
Trigonometric reminds

$$\cos^2(a) = \frac{1}{2} + \frac{1}{2}\cos(2a)$$

$$\sin^2(a) = \frac{1}{2} - \frac{1}{2}\cos(2a)$$

$$\sin(a)\cos(a) = \frac{1}{2}\sin(2a)$$

Homodyne demodulation



Upper branch
(in phase component)

$$2A_n \cos\left(2\pi f_n t+\varphi_n\right)\cos\left(2\pi f_n t\right)=$$

$$=2A_n \cos\left(2\pi f_n t\right)\left[\cos\left(2\pi f_n t\right)\cos\left(\varphi_n\right)-\sin\left(2\pi f_n t\right)\sin\left(\varphi_n\right)\right]=$$

$$=2A_n \cos\left(\varphi_n\right)\cos^2\left(2\pi f_n t\right)-2A_n \sin\left(2\pi f_n t\right)\cos\left(2\pi f_n t\right)\sin\left(\varphi_n\right)=$$

$$=2A_n \cos\left(\varphi_n\right)\left[\frac{1}{2}+\frac{1}{2}\cos\left(4\pi f_n t\right)\right]-A_n \sin\left(\varphi_n\right)\sin\left(4\pi f_n t\right)\Rightarrow$$

after filtering we achieve $A_n \cos\left(\varphi_n\right)=I_n$

Lower branch (quadrature component)

$$2A_n \cos\left(2\pi f_n t + \varphi_n\right)\sin\left(2\pi f_n t\right) =$$

$$= 2A_n \sin\left(2\pi f_n t\right)\left[\cos\left(2\pi f_n t\right)\cos\left(\varphi_n\right) - \sin\left(2\pi f_n t\right)\sin\left(\varphi_n\right)\right] =$$

$$= A_n \cos\left(\varphi\right)\sin\left(4\pi f_n t\right) - 2A_n \sin^2\left(2\pi f_n t\right)\sin\left(\varphi_n\right) =$$

$$= A_n \cos\left(\varphi_n\right)\sin\left(4\pi f_n t\right) - 2A_n \left[\frac{1}{2} - \frac{1}{2}\cos\left(4\pi f_n t\right)\right]\sin\left(\varphi_n\right) \Rightarrow$$

$$\textit{after} \quad \textit{filtering we achieve} \quad -A_n \sin\left(\varphi\right) = Q_n$$

Final result for each harmonic signal

$$h_n(t) = I_n \cos(2\pi f_n t) + Q_n \sin(2\pi f_n t) =$$

$$= A_n \cos(2\pi f_n t + \varphi_n)$$

Synthetic pulse

$$s(t) = \sum_{n=1}^{N} h_n(t)$$

Drawbacks of the homodyne demodulation

The baseband signal is subject to more noise, in particular the flicker noise, approximately decreasing as 1/f or as a roof function.

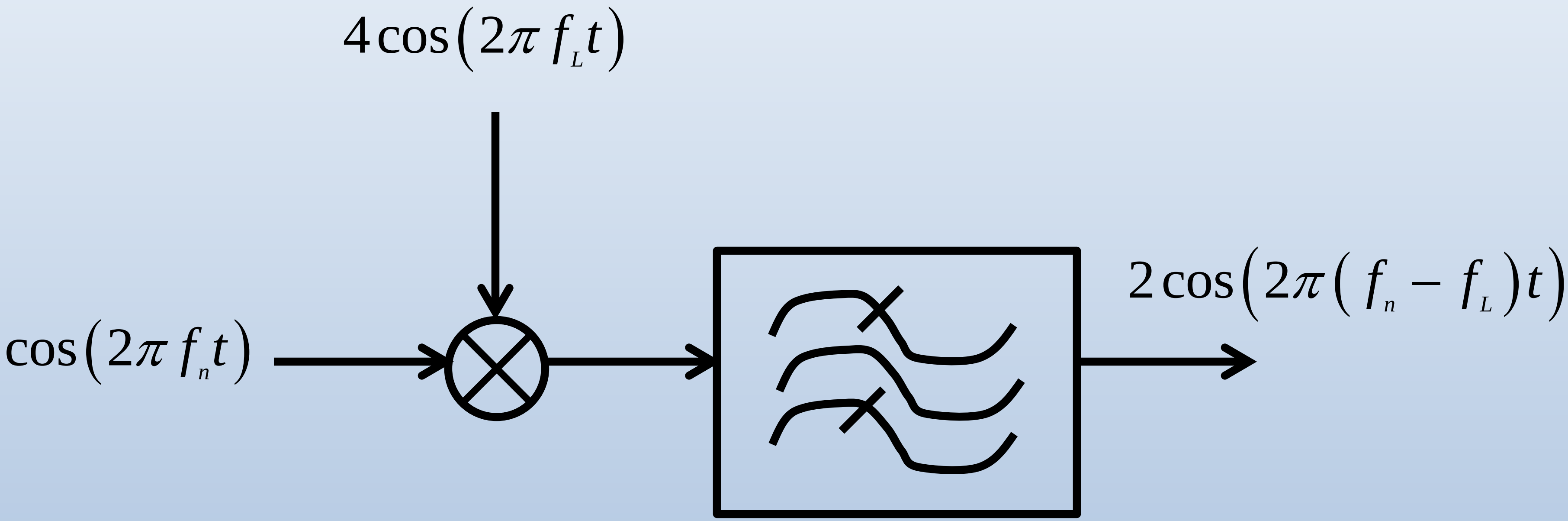
It is difficult to translate in baseband N signals from N different frequencies keeping uniformly low the noise.

For this reason it usually preferred an heterodyne demodulation scheme.

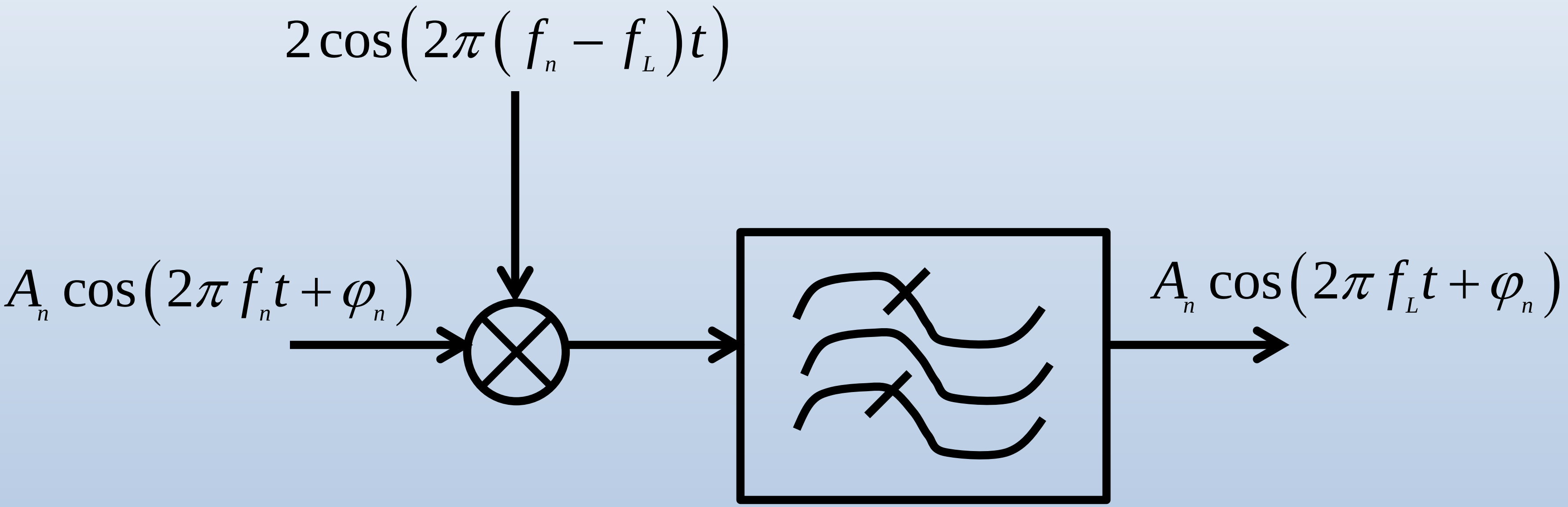
Trigonometric reminds

$$\begin{cases} 2\cos(a)\cos(b) = \cos(a+b) + \cos(a-b) \\ 2\sin(a)\cos(b) = \sin(a+b) + \sin(a-b) \end{cases}$$

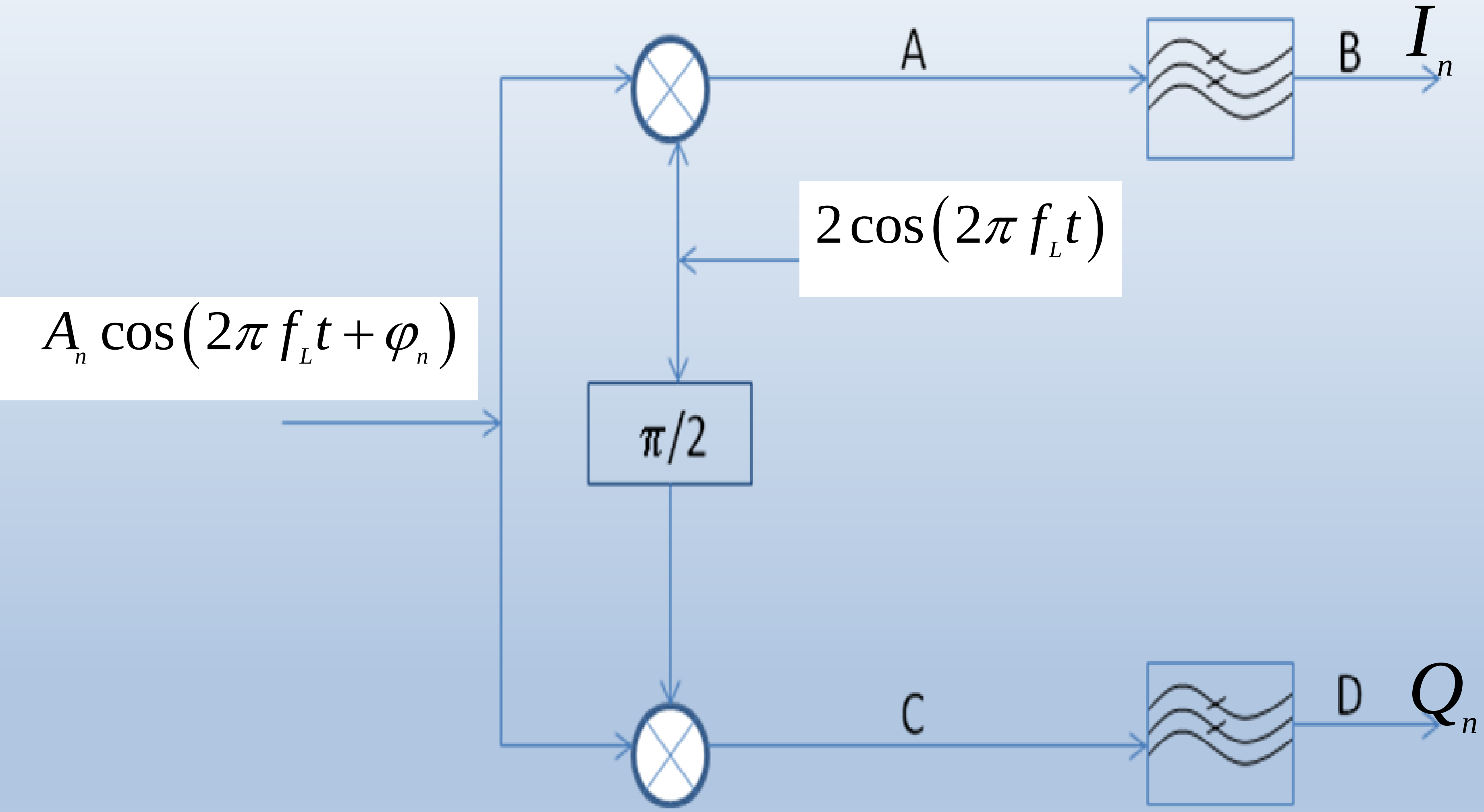
Translation of the radiated signal on the frequency axis



Translation of the received signal on the frequency axis



Heterodyne demodulation



Averaged measurement of the in-phase and quadrature components

$$\begin{cases} I_m = \dfrac{I_{1m} + I_{2m} + \ldots I_{Nm}}{N} \\ Q_m = \dfrac{Q_{1m} + Q_{2m} + \ldots Q_{Nm}}{N} \end{cases}$$

(N is chosen by the manufacturer)

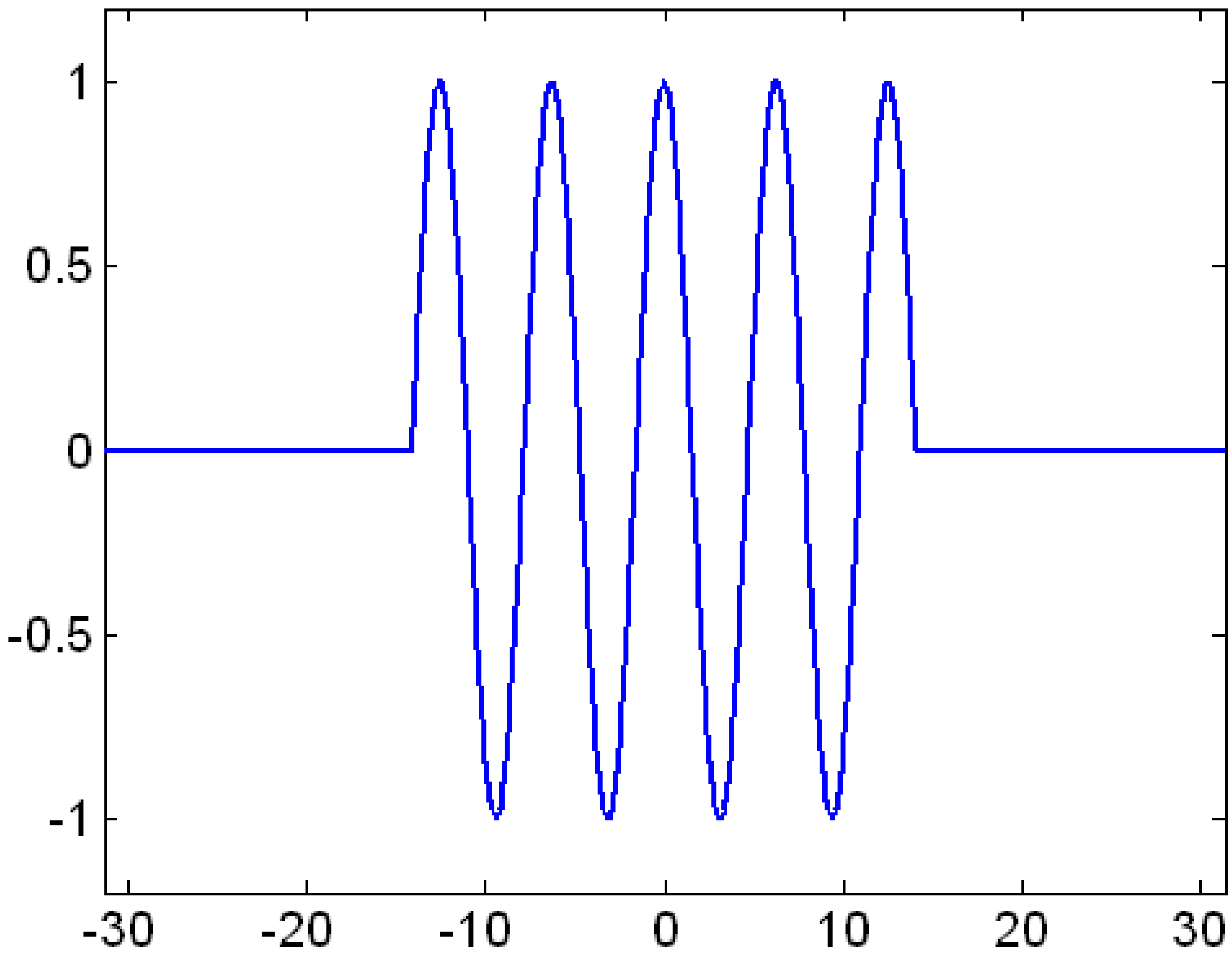
Power of the noise (white noise) on each received “harmonic” signal

$$N = K_B TB$$

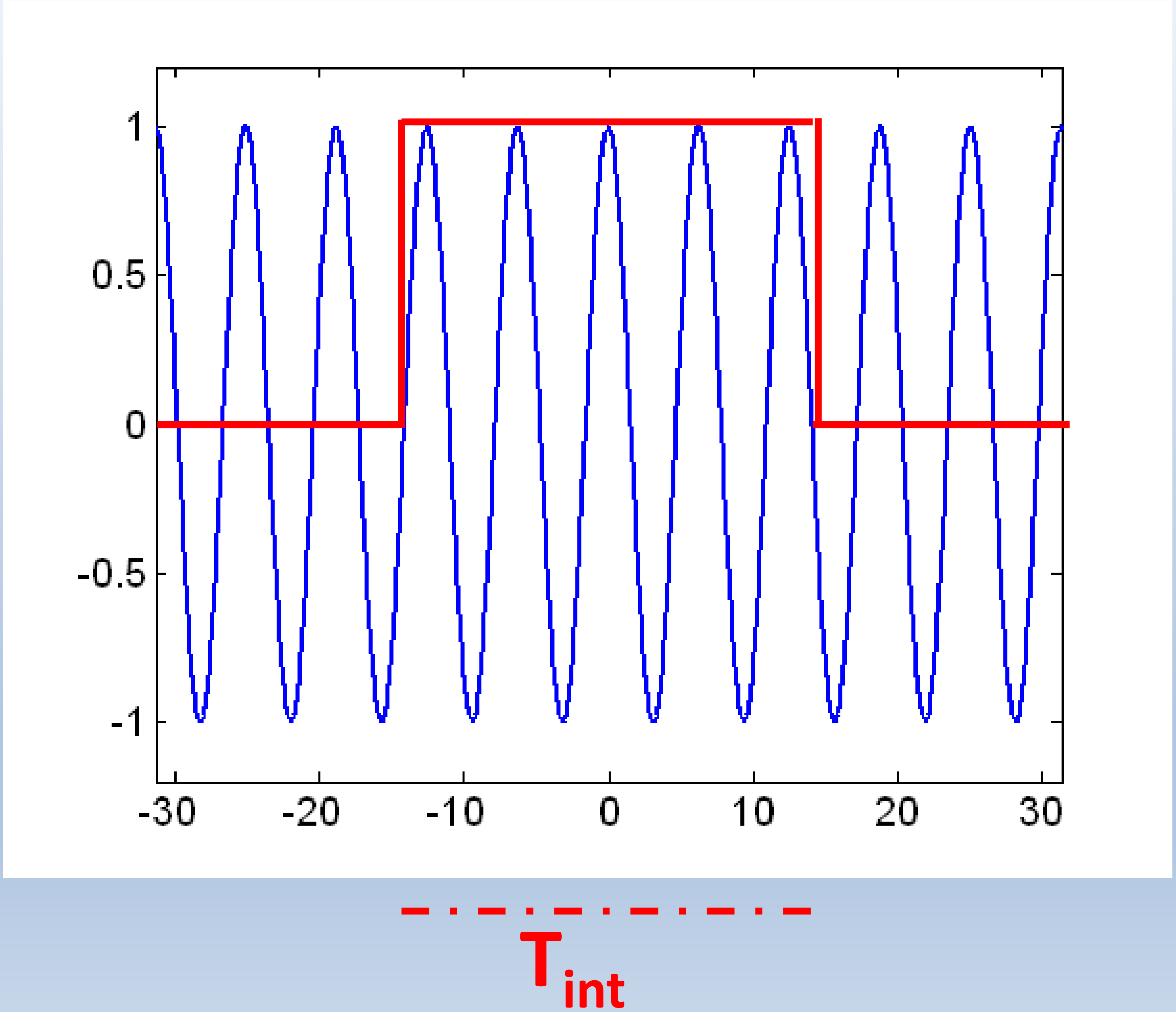
$$K_B = 1.38 \times 10^{-23} \ \frac{J}{K^o}$$

$$SNR \approx \frac{\left\| S(f) \right\|^2}{K_B TB}$$

Truncated sinusoid



Integration time



Spectrum of any radiated truncated harmonic signal

$$s_n(t) = \cos(2\pi f_n t)\,\Pi\left(\frac{t}{T_{\text{int}}}\right)$$

$$FT\left(\Pi\left(\frac{t}{T_{\text{int}}}\right)\right) = T_{\text{int}}\,\text{sinc}(\pi f T_{\text{int}})$$

$$FT\left(\cos(2\pi f_n t)\right) = \frac{1}{2}\left\{\delta(f - f_n) + \delta(f + f_n)\right\}$$

$$S_n(f) = \frac{T_{\text{int}}}{2}\left\{\text{sinc}\left(\pi T_{\text{int}}(f - f_n)\right) + \text{sinc}\left(\pi T_{\text{int}}(f + f_n)\right)\right\}$$

Let us consider the "pieces"

$$\mathrm{sinc}\left(\pi T_{\mathrm{int}}\left(f - f_n\right)\right), \mathrm{sinc}\left(\pi T_{\mathrm{int}}\left(f + f_n\right)\right)$$

The main lobe is wide $1/T_{int}$ for both pieces

$$SNR \approx \frac{\left\|S_n\left(f\right)\right\|^2}{K_B T B_n} \approx \frac{T_{\mathrm{int}}\left\|S_n\left(f\right)\right\|^2}{K_B T}$$

Prolonging the integration time the SNR increases, but the measurement requires more time

- ***The integration time is a default parameter chosen by the manufacturer.***

- ***Possibly, there is the possibility to extend the default integration time times a factor.***

- ***The integration time is usually the same for all the harmonics spanned by the stepped frequency system.***

NON-AMBIGUOUS TIME INTERVAL

The time windows where we can reliably examine the synthetic pulses is given by

$$T_{\max} = \frac{1}{\Delta f}$$

The depth investigated cannot exceed the non-ambiguous level

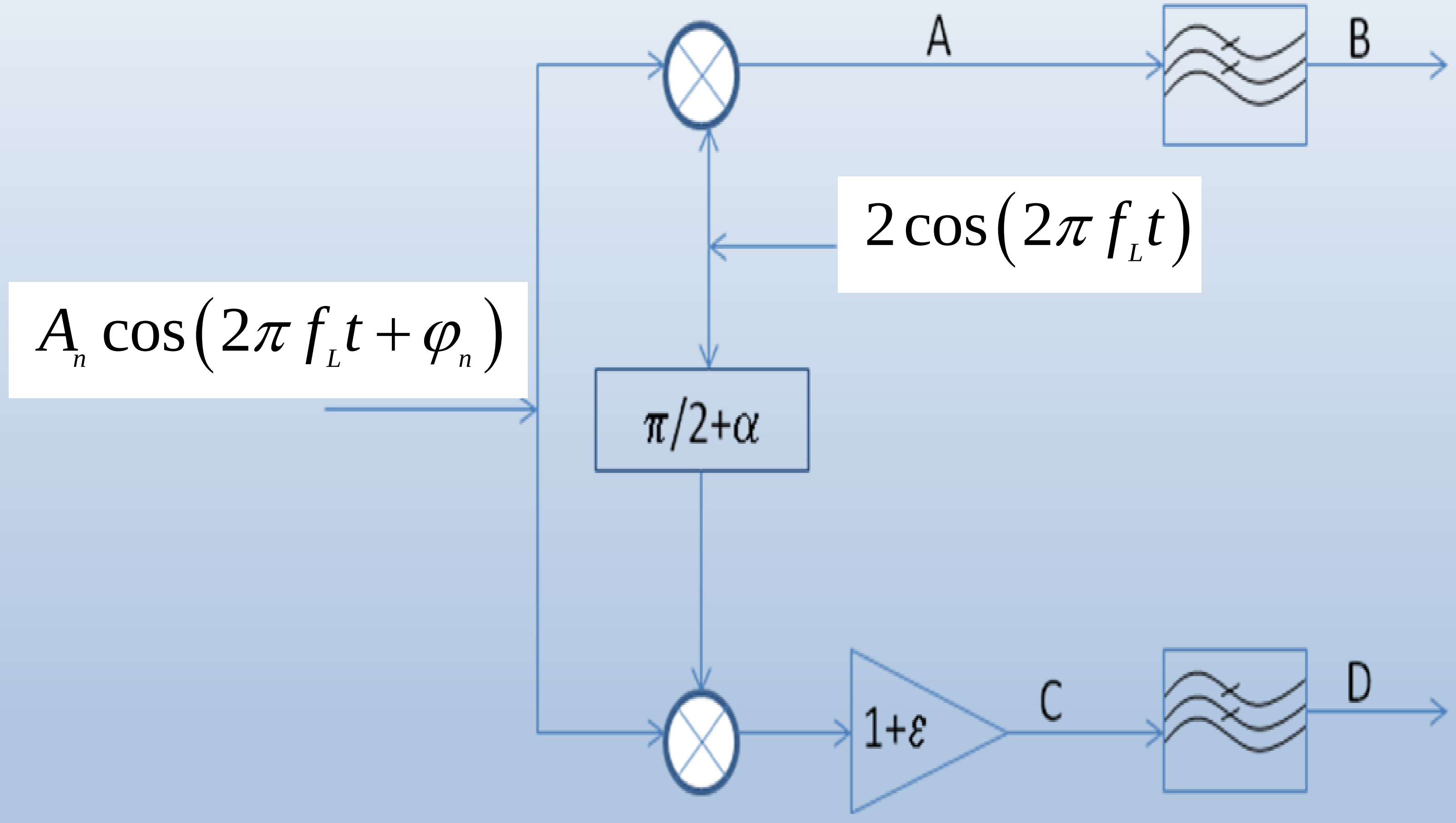
$$D_{\max} = \frac{cT_{\max}}{2} = \frac{c}{2\Delta f}$$

The frequency step cannot exceed the maximum value:

$$\Delta f \leq \frac{c}{2D_{\max}} = \frac{c_o}{2\sqrt{\varepsilon_r \mu_r} D_{\max}}$$

N.B.: D_{max} depends on the penetration of the signal, not on the maximum depth of interest.

Effect of possible imperfections on the demodulation chain



Received synthetic pulse for a target at depth level t_o (the spectrum of the signal is considered flat in its band, sampled with 2N+1 frequencies)

$$s_r(t) \approx K \times$$

$$\times \left\{ \sqrt{\left(1+\frac{\varepsilon}{2}\right)^2 + \frac{\alpha^2}{4}} \cos\left(2\pi f_c (t-t_o) + \theta - tg^{-1}\frac{\alpha}{2+\varepsilon}\right) \frac{\sin\left((2N+1)\pi\Delta f\,(t-t_o)\right)}{\sin\left(\pi\Delta f\,(t-t_o)\right)} + \right.$$

$$\left. -\sqrt{\frac{\varepsilon^2}{4} + \frac{\alpha^2}{4}} \cos\left(2\pi f_c (t+t_o) - \theta - tg^{-1}\frac{\alpha}{\varepsilon}\right) \frac{\sin\left((2N+1)\pi\Delta f\,(t+t_o)\right)}{\sin\left(\pi\Delta f\,(t+t_o)\right)} \right\}$$

Hermitian image at $\frac{1}{\Delta f} - t_o$

Signal to Hermitian image ratio SHR

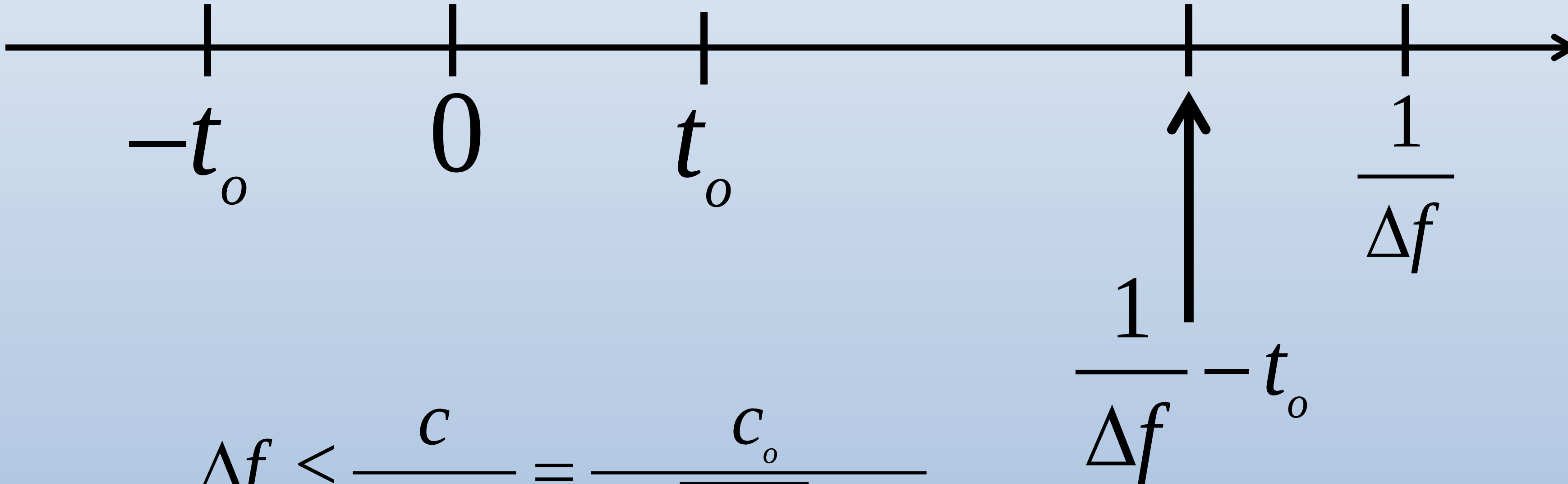
$$s_r(t) \approx K \times$$

$$\times \left\{ \sqrt{\left(1+\frac{\varepsilon}{2}\right)^2 + \frac{\alpha^2}{4}} \cos\left(2\pi f_c\left(t-t_o\right)+\theta - tg^{-1}\frac{\alpha}{2+\varepsilon}\right) \frac{\sin\left(\left(2N+1\right)\pi\Delta f\left(t-\overline{t}\right)\right)}{\sin\left(\pi\Delta f\left(t-\overline{t}\right)\right)} + \right.$$

$$\left. -\sqrt{\frac{\varepsilon^2}{4}+\frac{\alpha^2}{4}} \cos\left(2\pi f_c\left(t+t_o\right)-\theta - tg^{-1}\frac{\alpha}{\varepsilon}\right) \frac{\sin\left(\left(2N+1\right)\pi\Delta f\left(t+\overline{t}\right)\right)}{\sin\left(\pi\Delta f\left(t+\overline{t}\right)\right)} \right\}$$

$$SHR \approx \frac{\sqrt{\left(1+\frac{\varepsilon}{2}\right)^2 + \frac{\alpha^2}{4}}}{\sqrt{\frac{\varepsilon^2}{4}+\frac{\alpha^2}{4}}} \approx \frac{2}{\sqrt{\varepsilon^2+\alpha^2}}$$

Counteraction against Hermitian images: halving the frequency step at parity of maximum investigated time depth

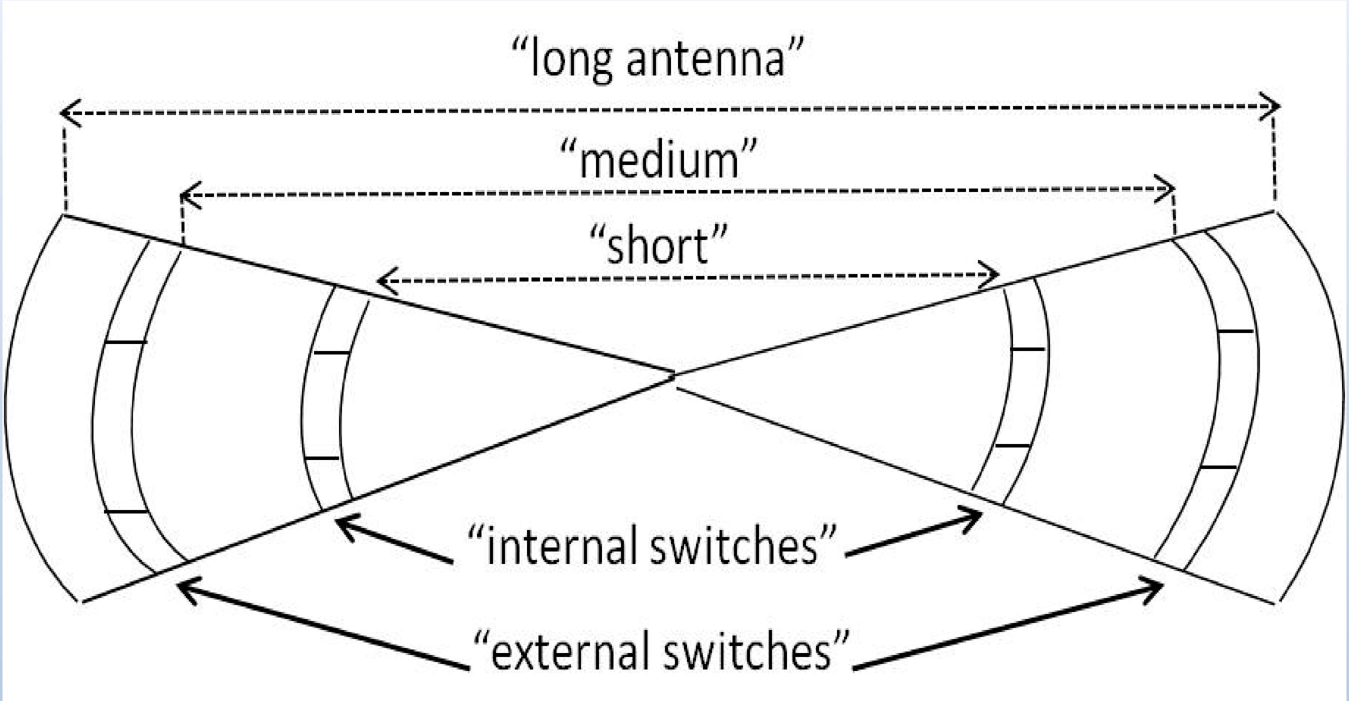


$$\Delta f \le \frac{c}{4D_{\max}} = \frac{c_o}{4\sqrt{\varepsilon_r \mu_r}\, D_{\max}}$$

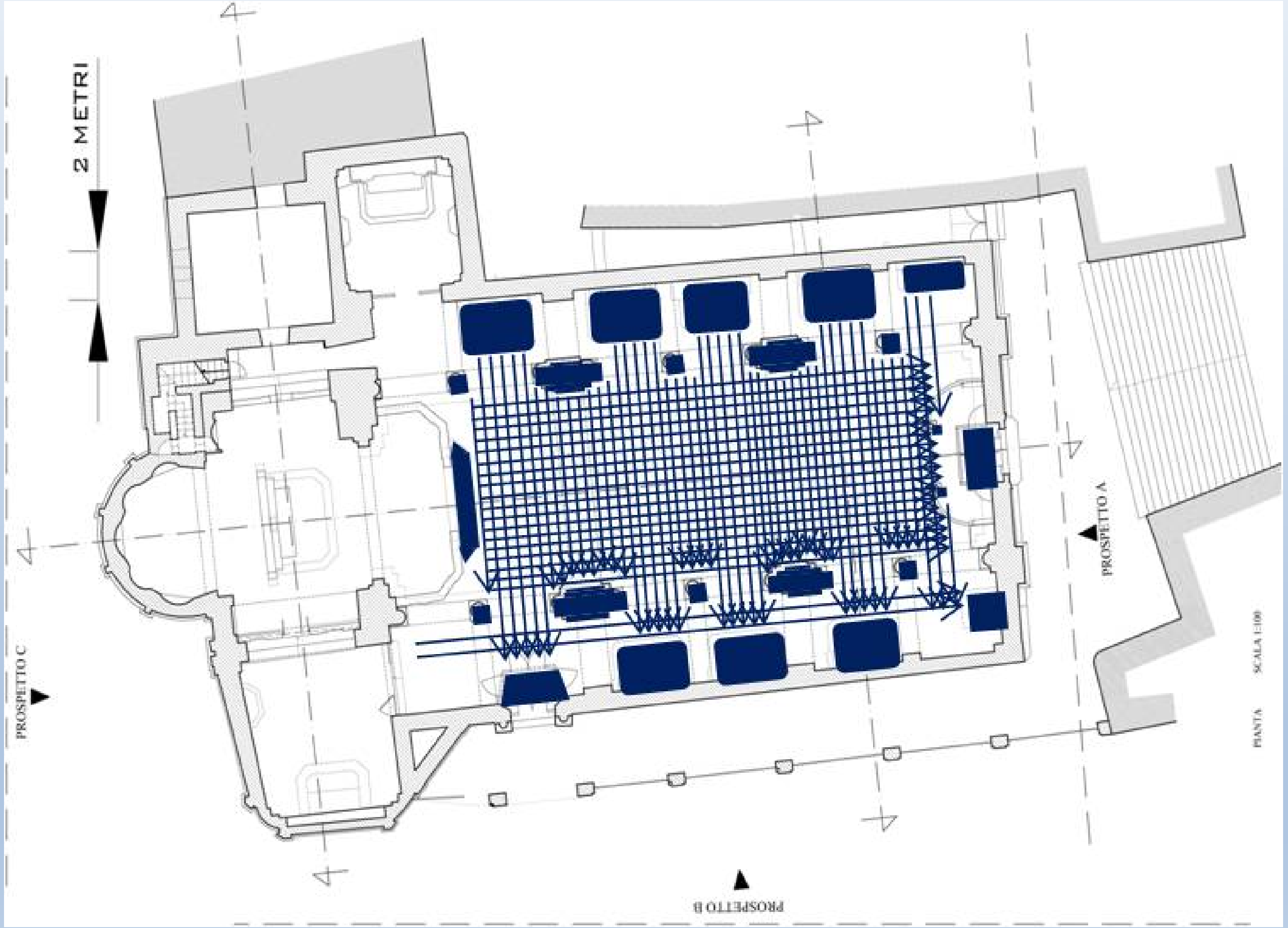
$$T_{\max} = \frac{1}{2\Delta f}$$

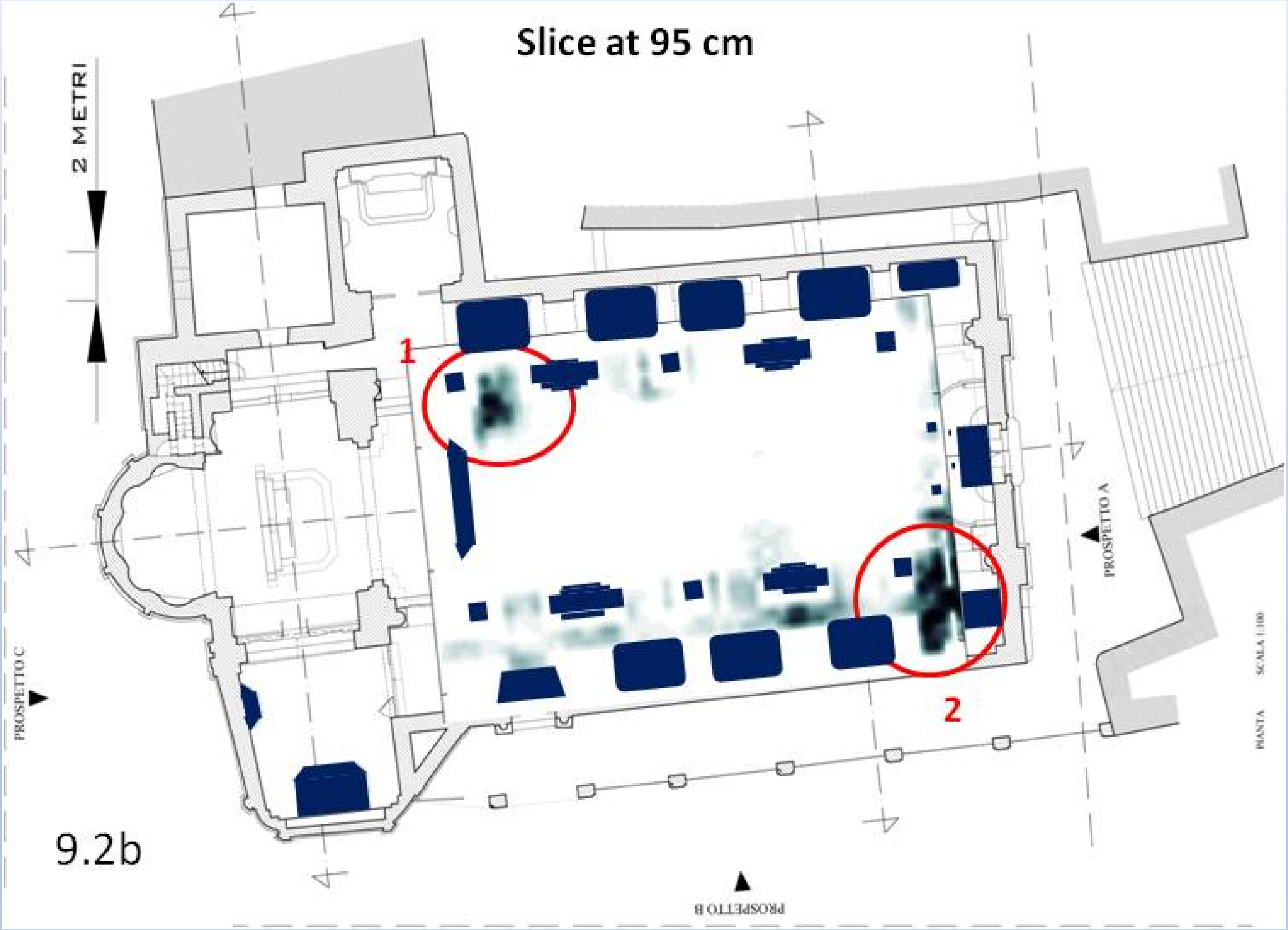
The reconfigurable GPR system (50 MHz-1 GHz)

Reconfigurable antennas

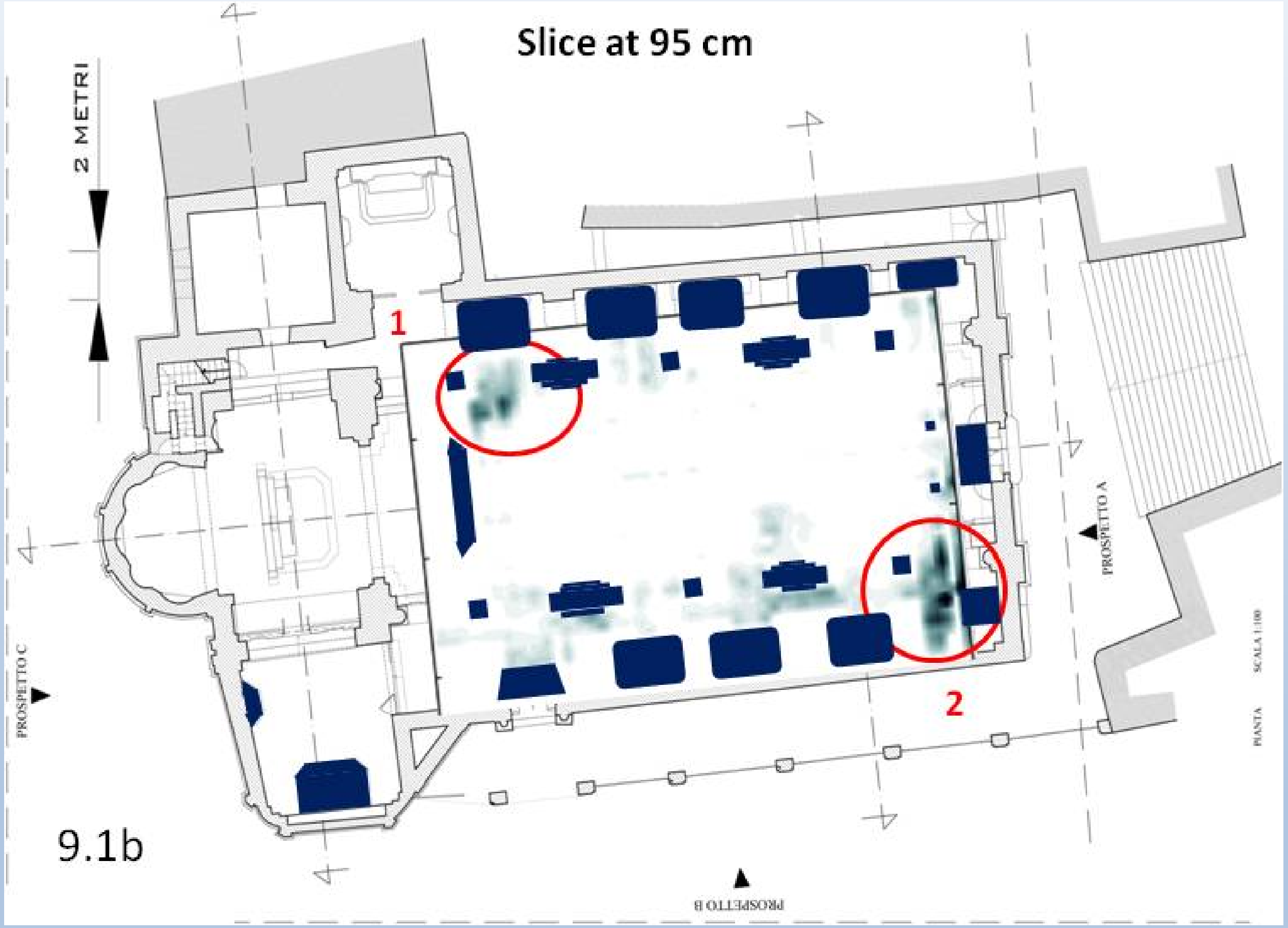


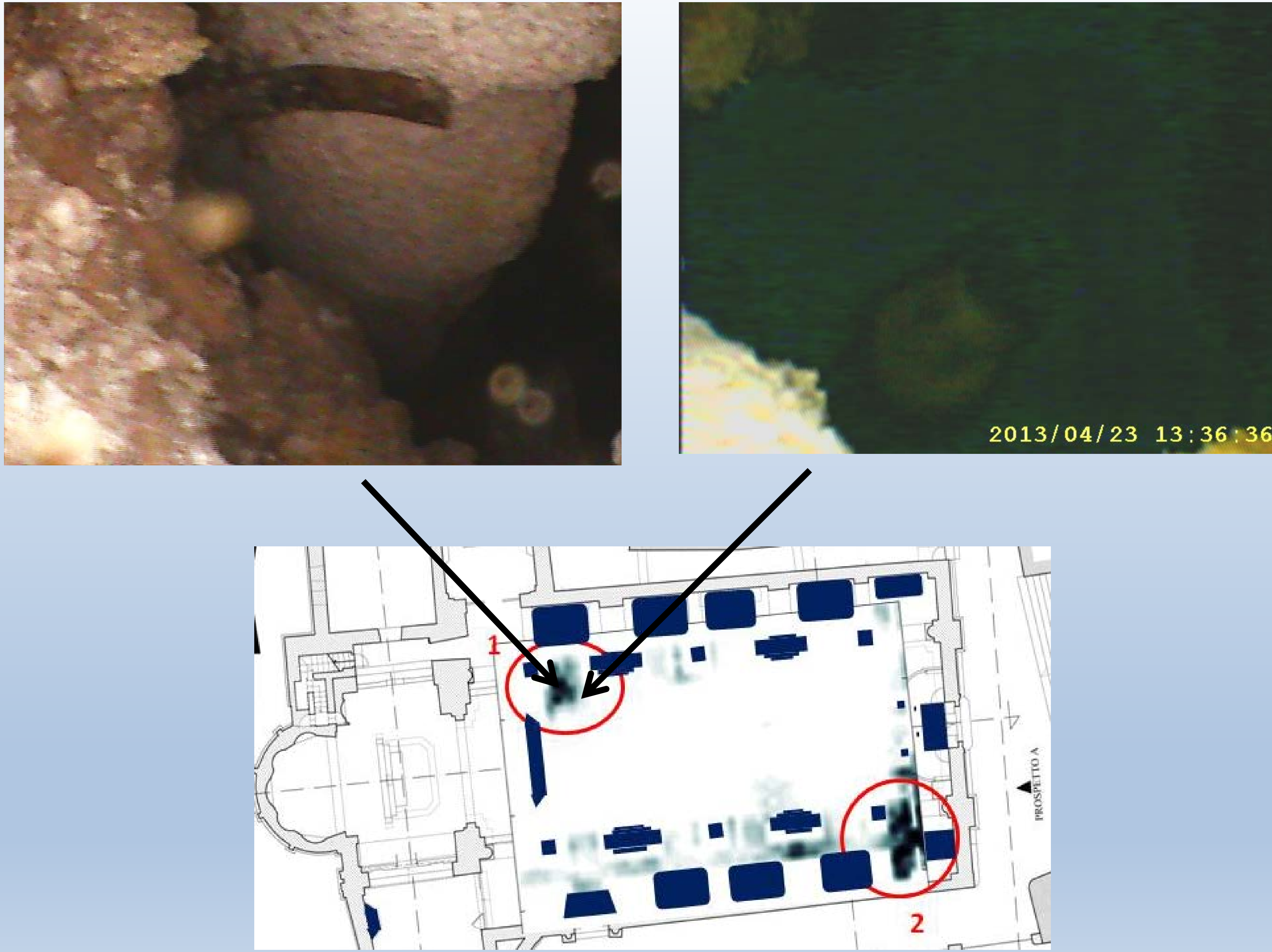
The matrix church of Parabita

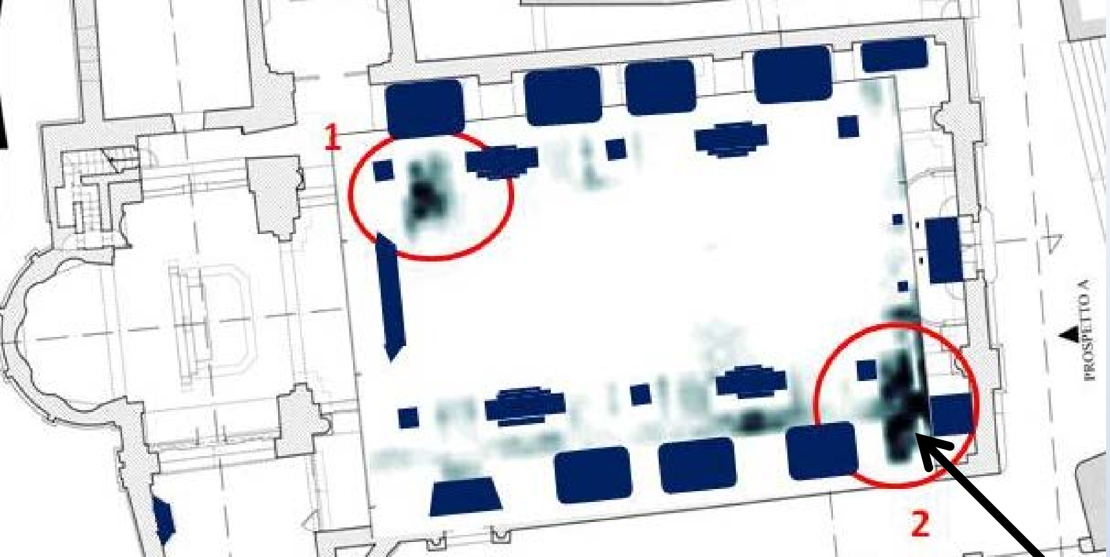



Slice at 95 cm
2 METRI
1
2
PROSPETTO A
PROSPETTO B
PROSPETTO C
9.2b

SLICE ACHIEVED WITH A RIS-HI MODE PULSED SYSTEM



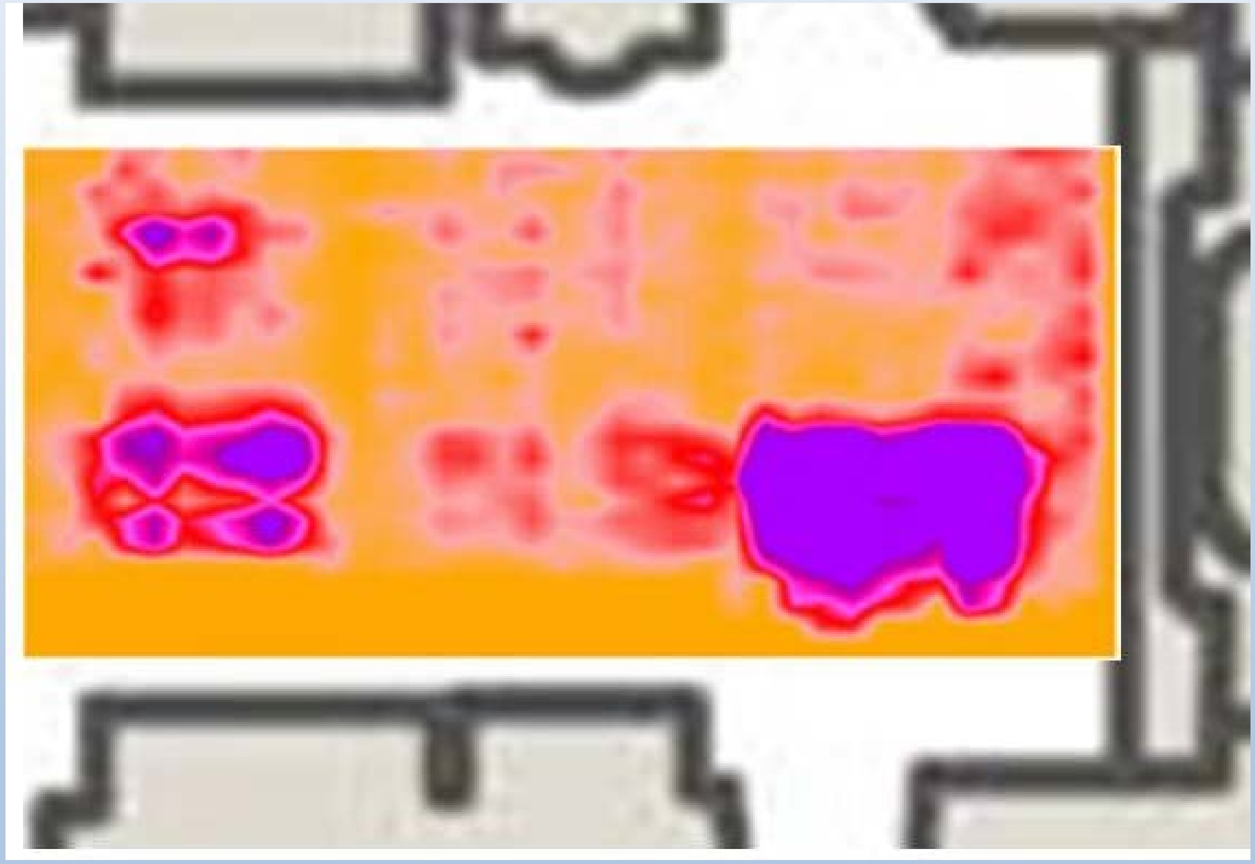

2013/04/23 13:36:36
1
2
PROSPETTO A


1
2
PROSPETTO A

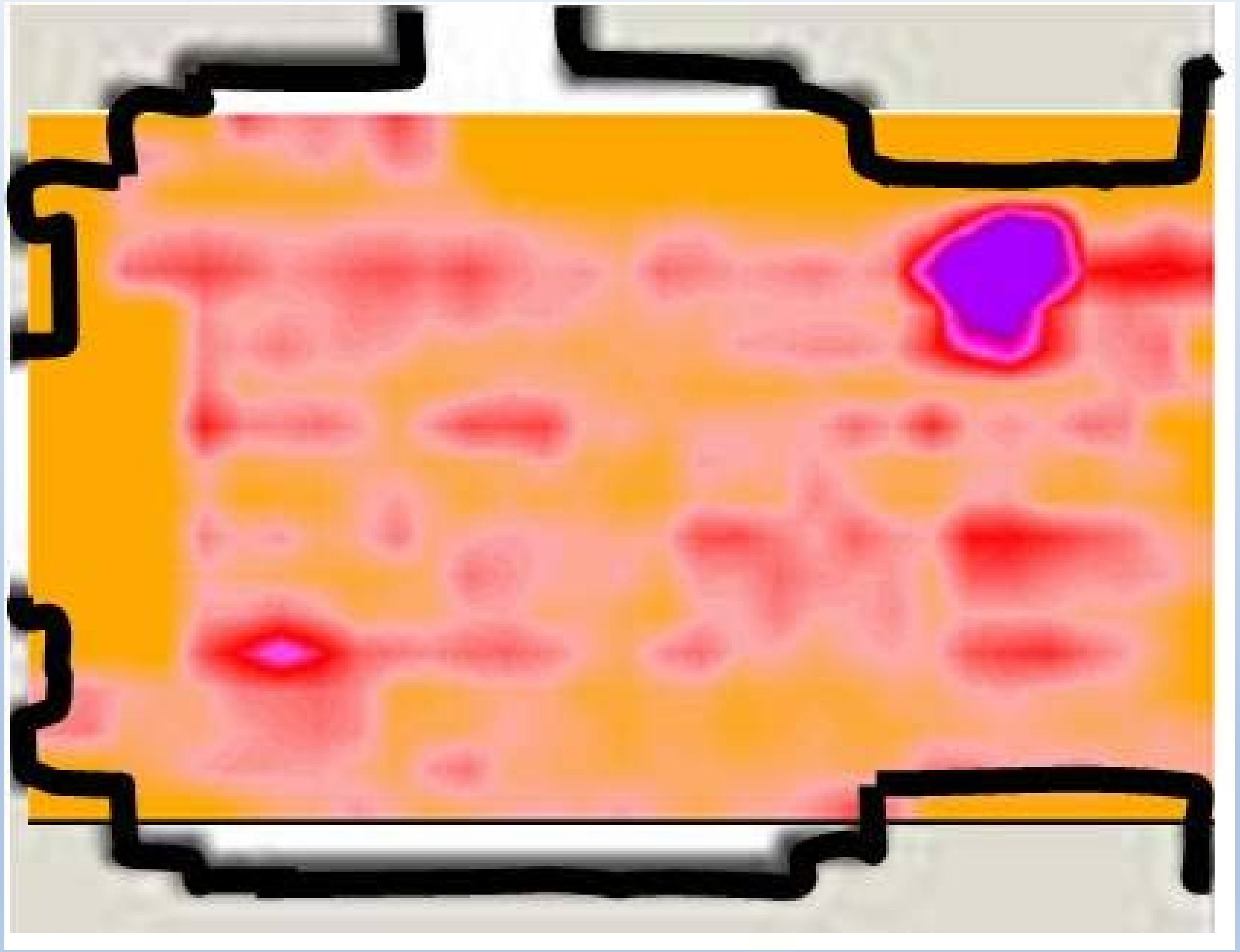

2013/04/23 14:14:06

The co-cathedral of St. John in Malta

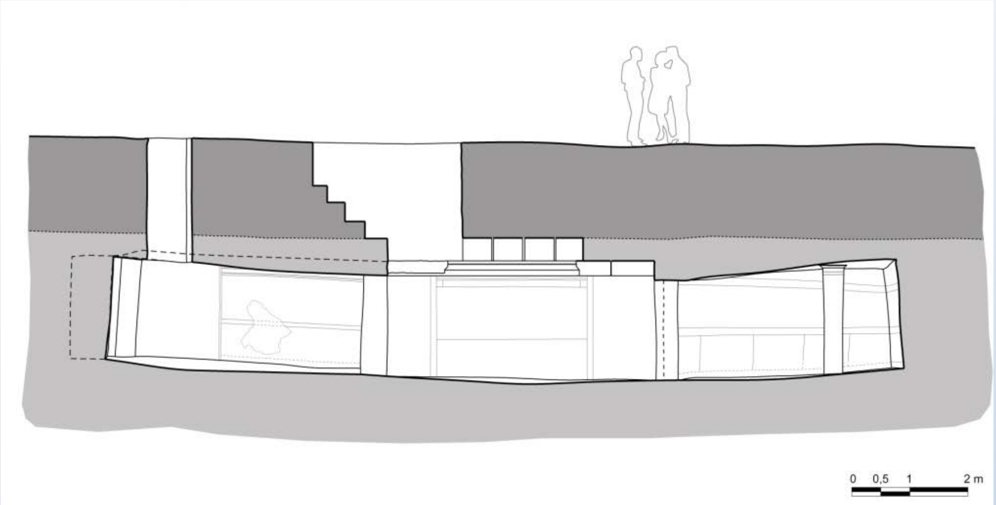
Chapel of Aragon

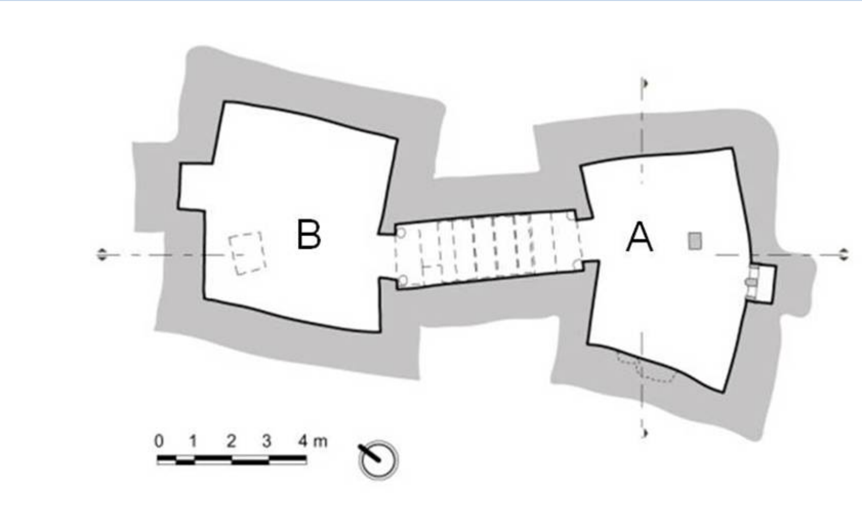


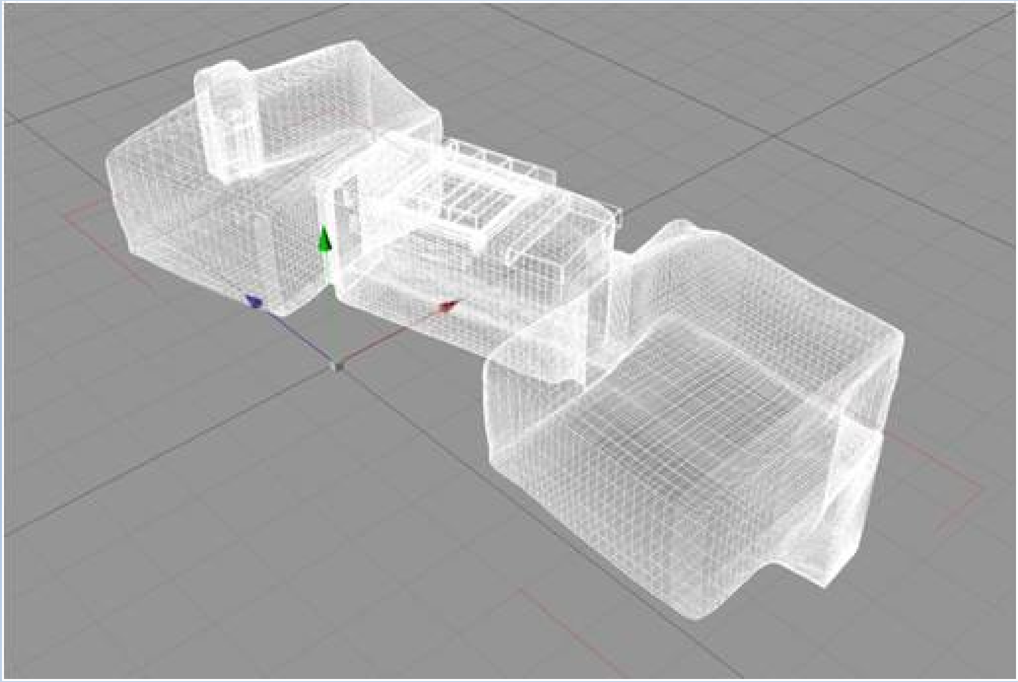
Chapel of the Sacristy

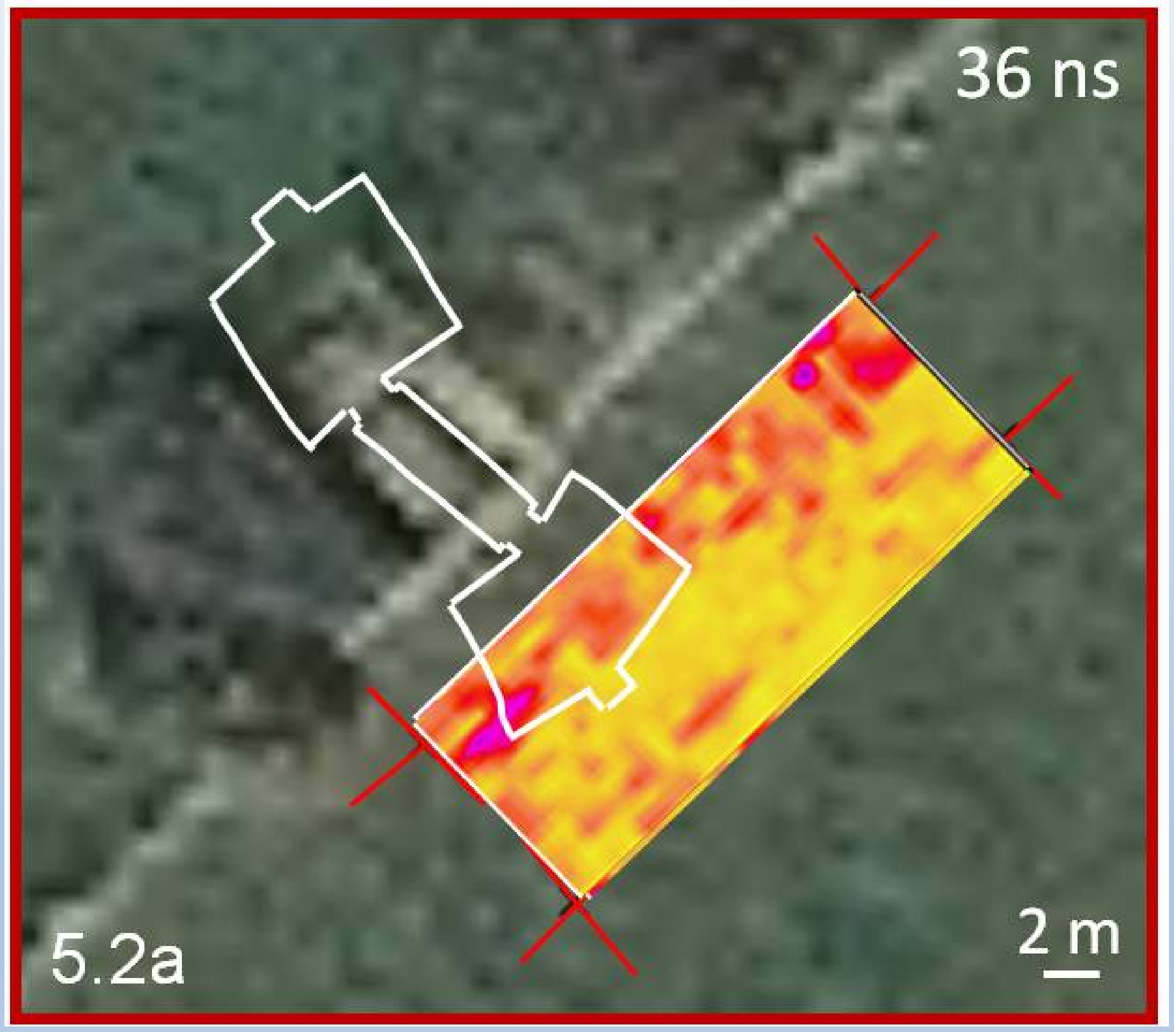


The Tomb of the Pillar

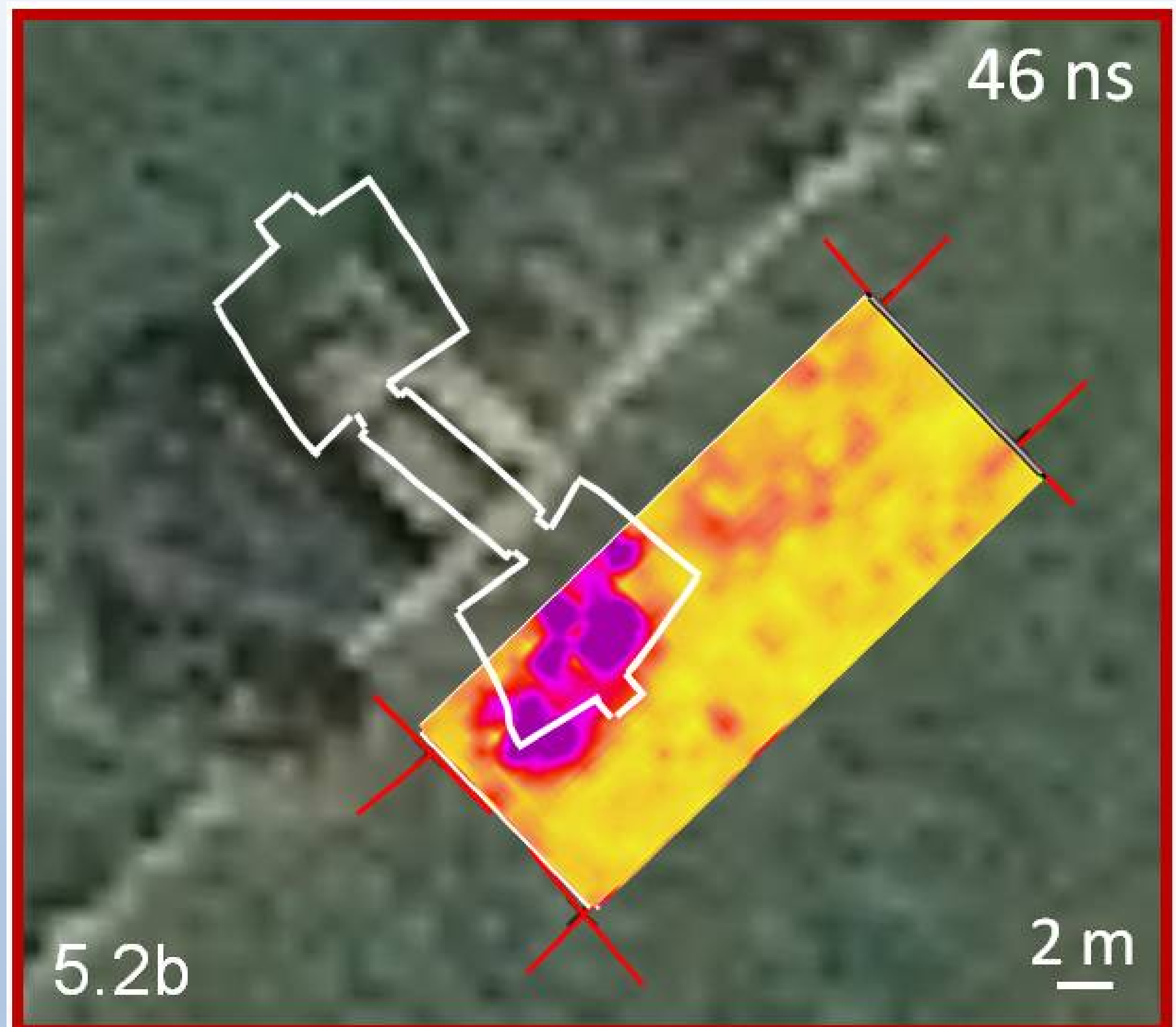




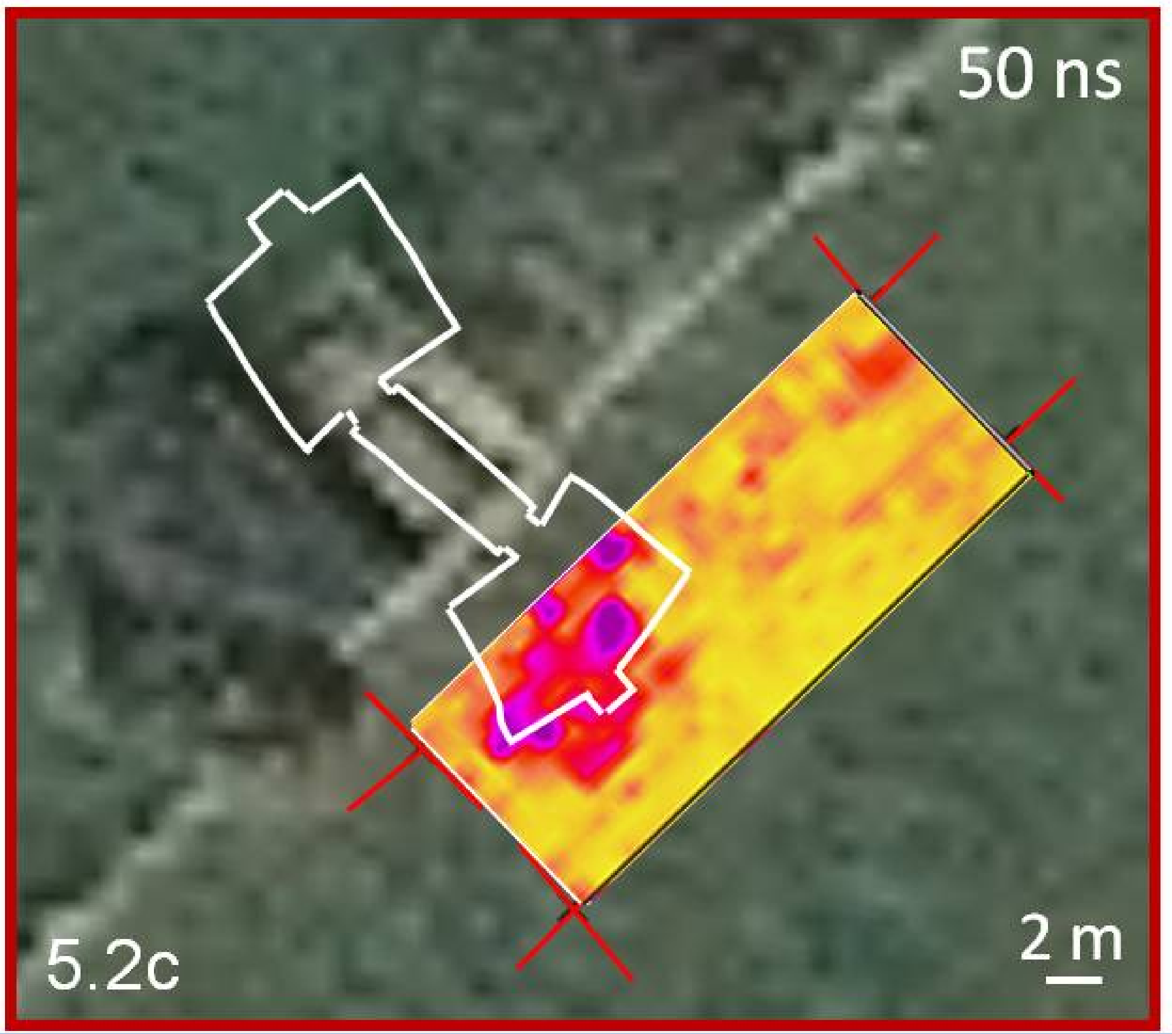




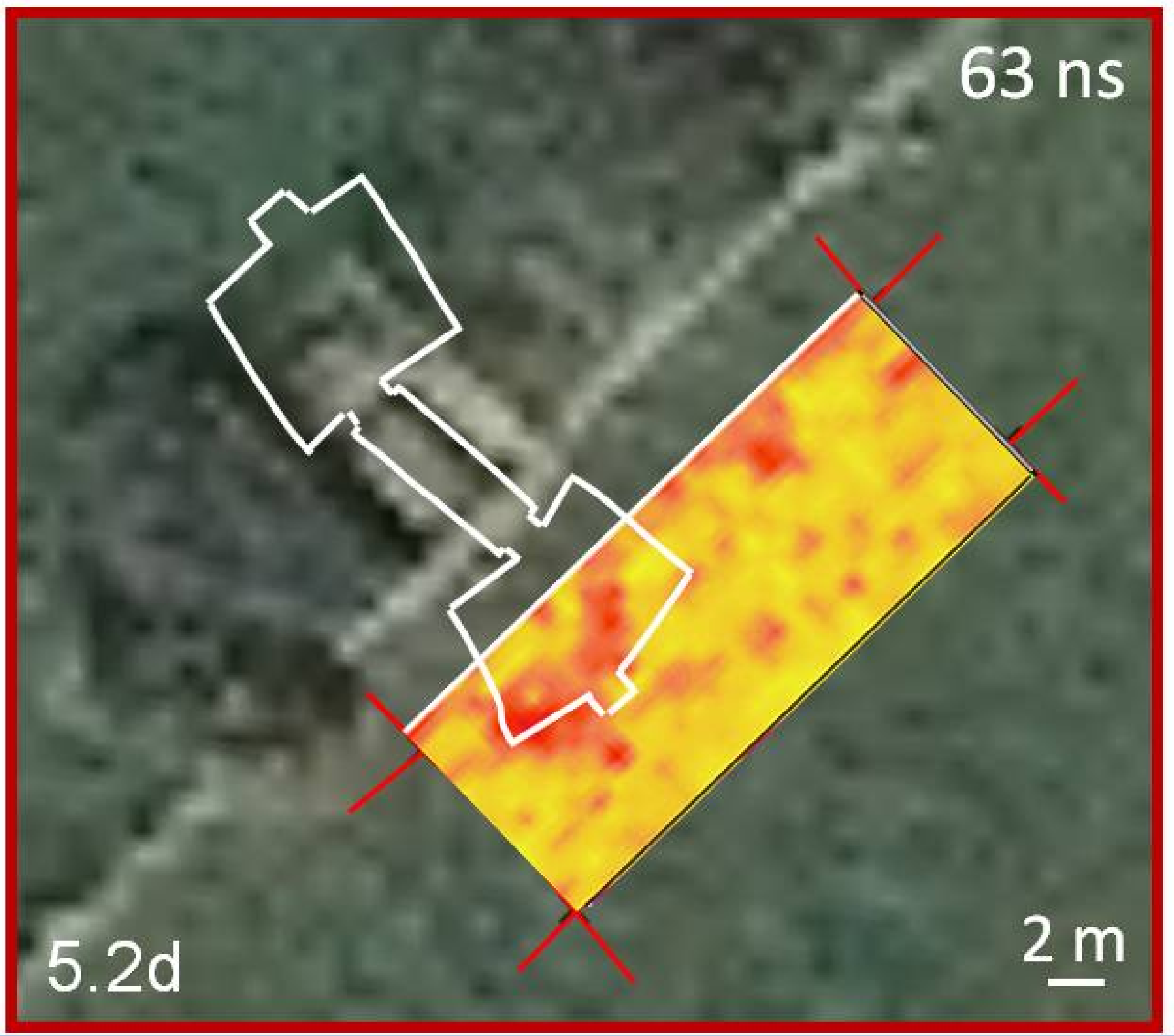
Depth 1.26 m



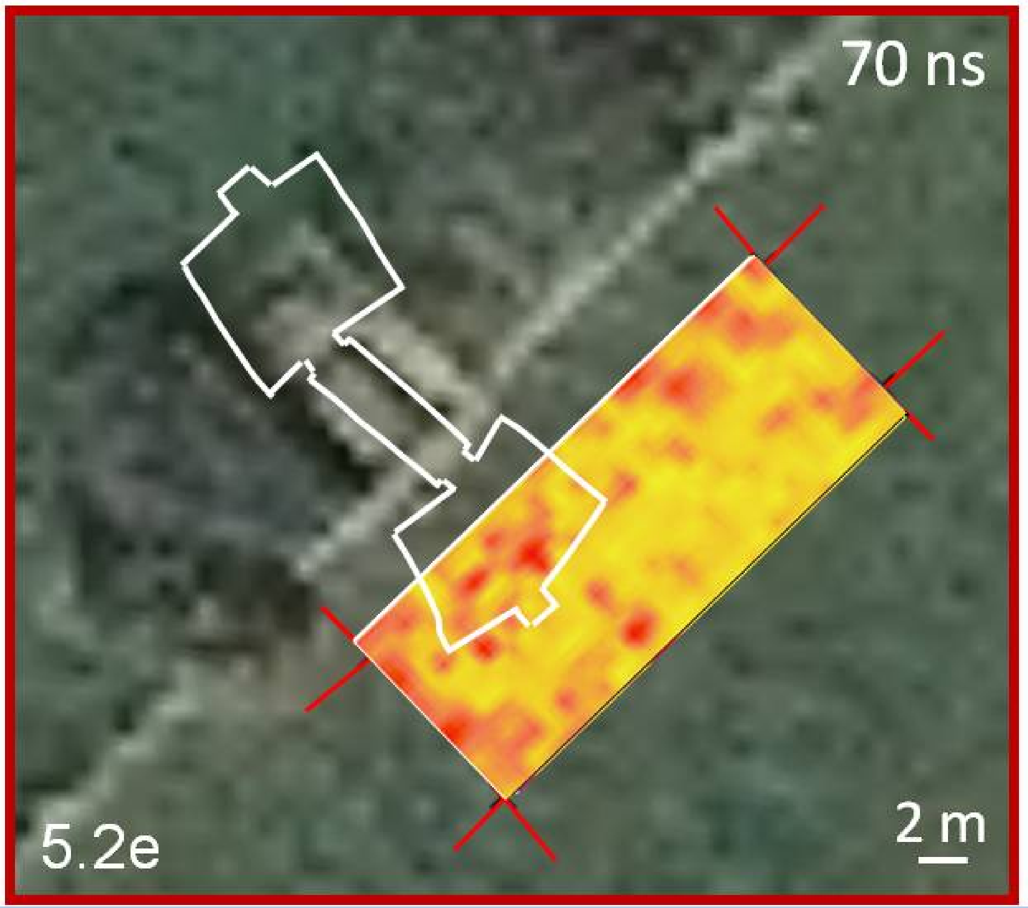
Depth 1.61m



Depth 1.75m



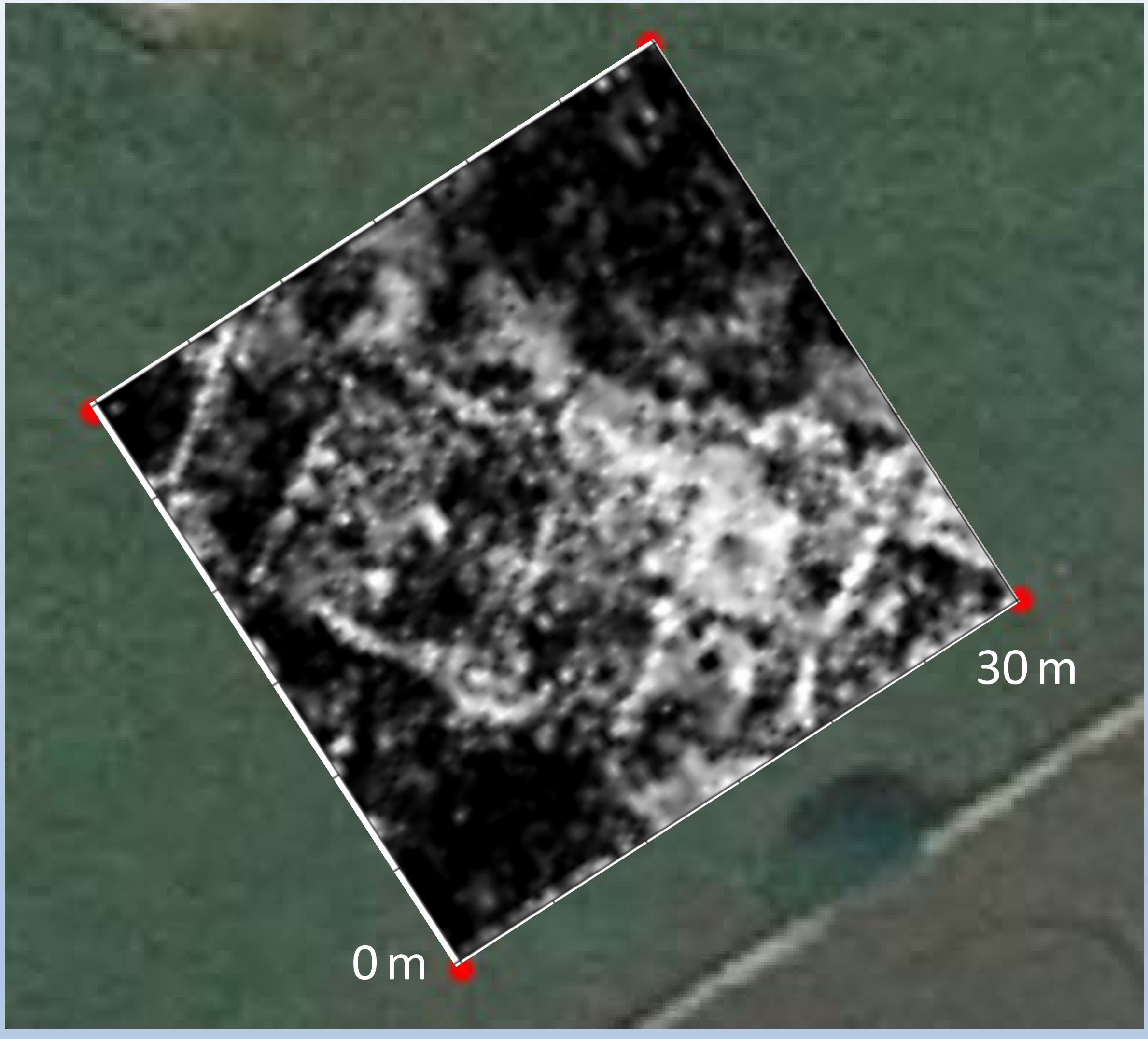
Depth 2.20m



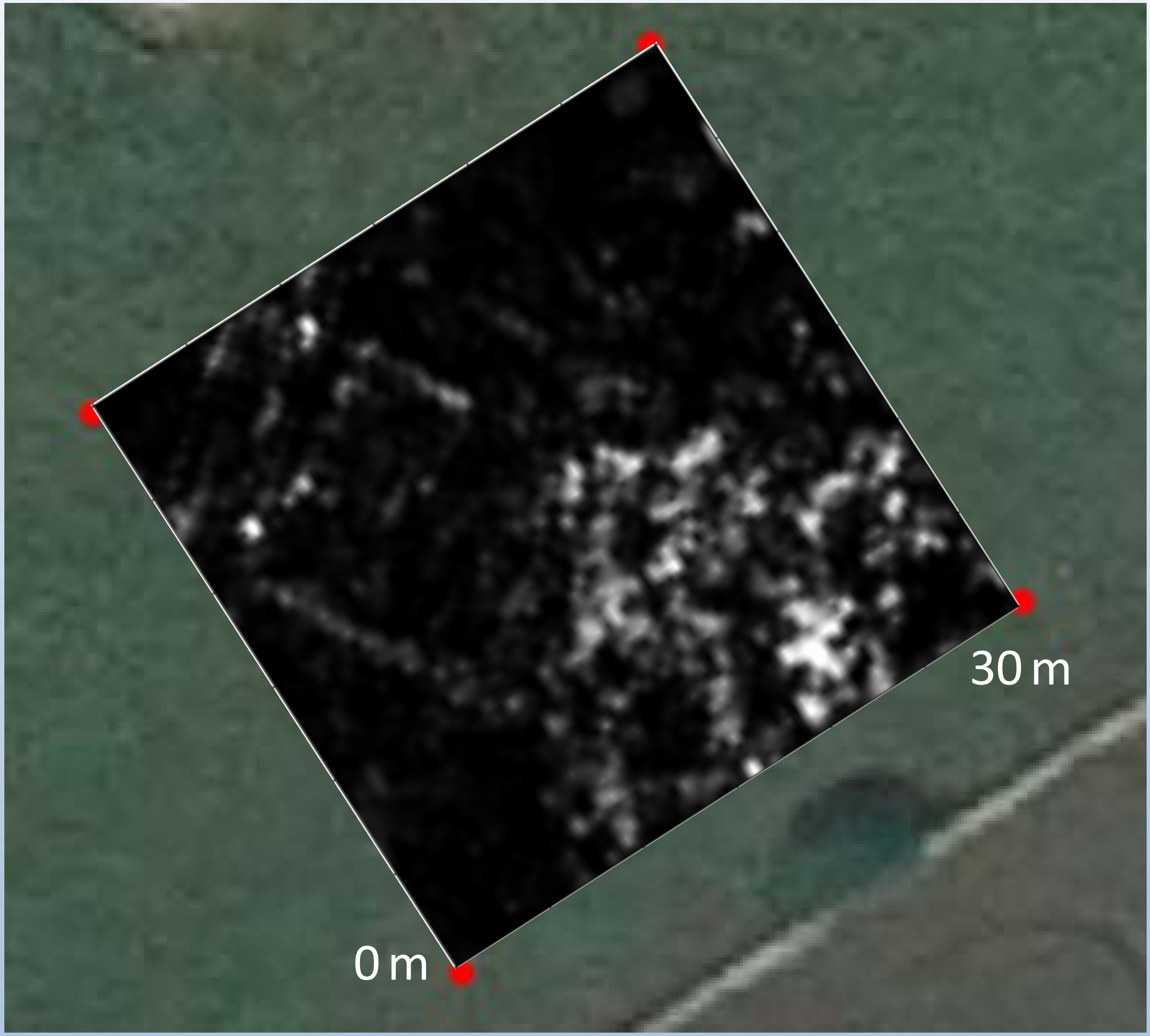
Depth 2.45m

Apparent thickness of the tomb 49 cm, real thickness 2.1 m

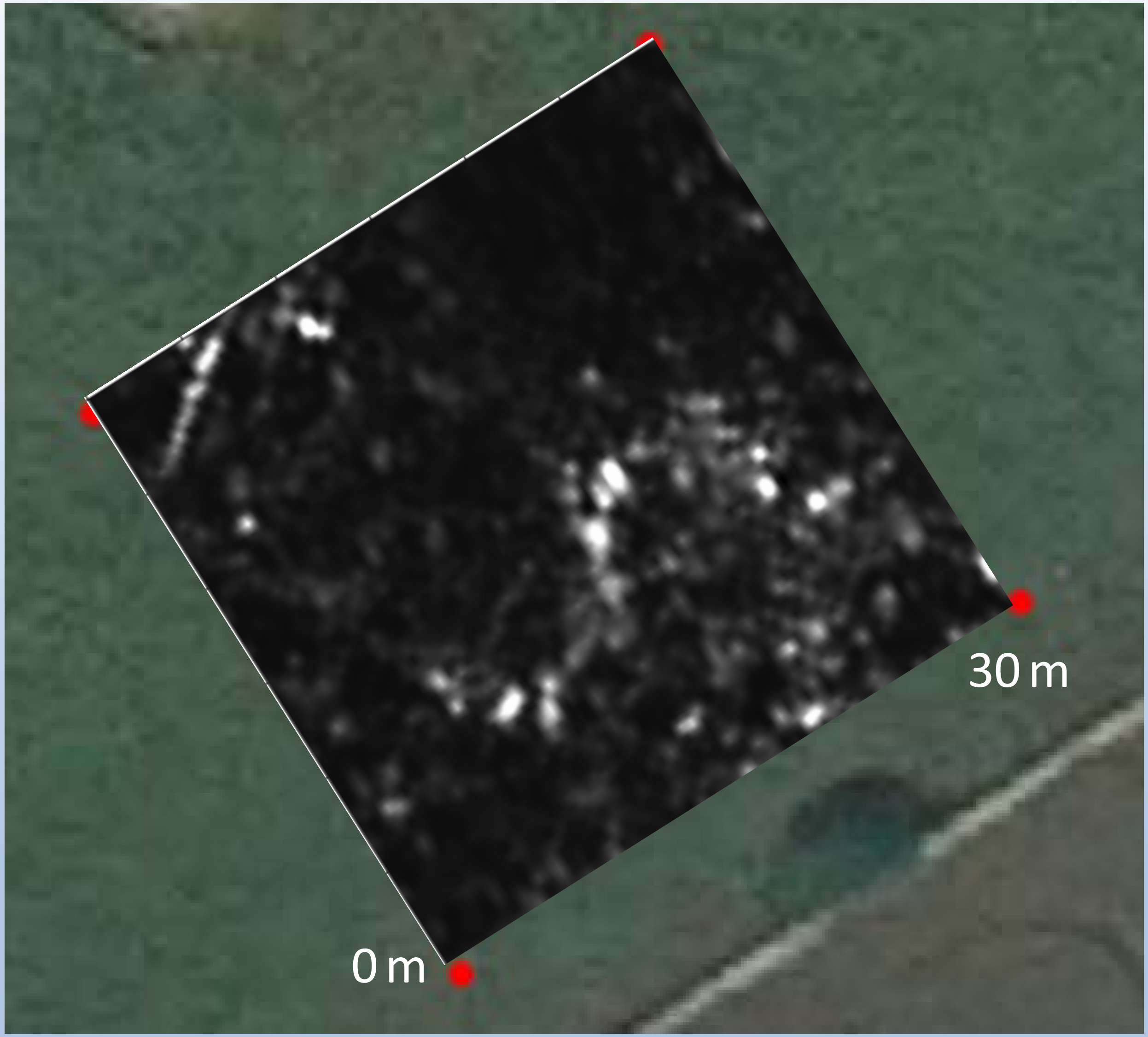
The area of the “cryptoporticus” in Egnazia



Depth 40 cm

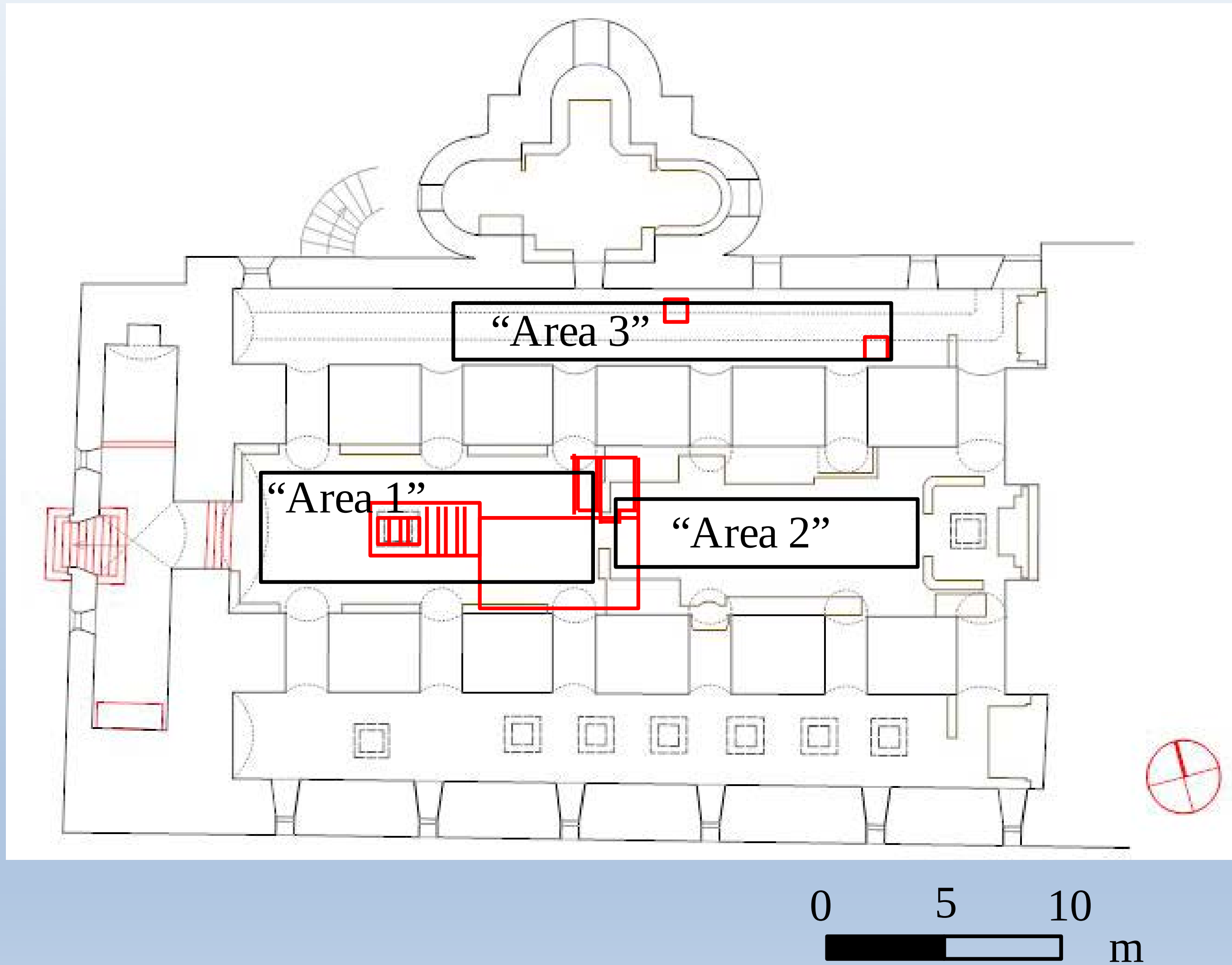


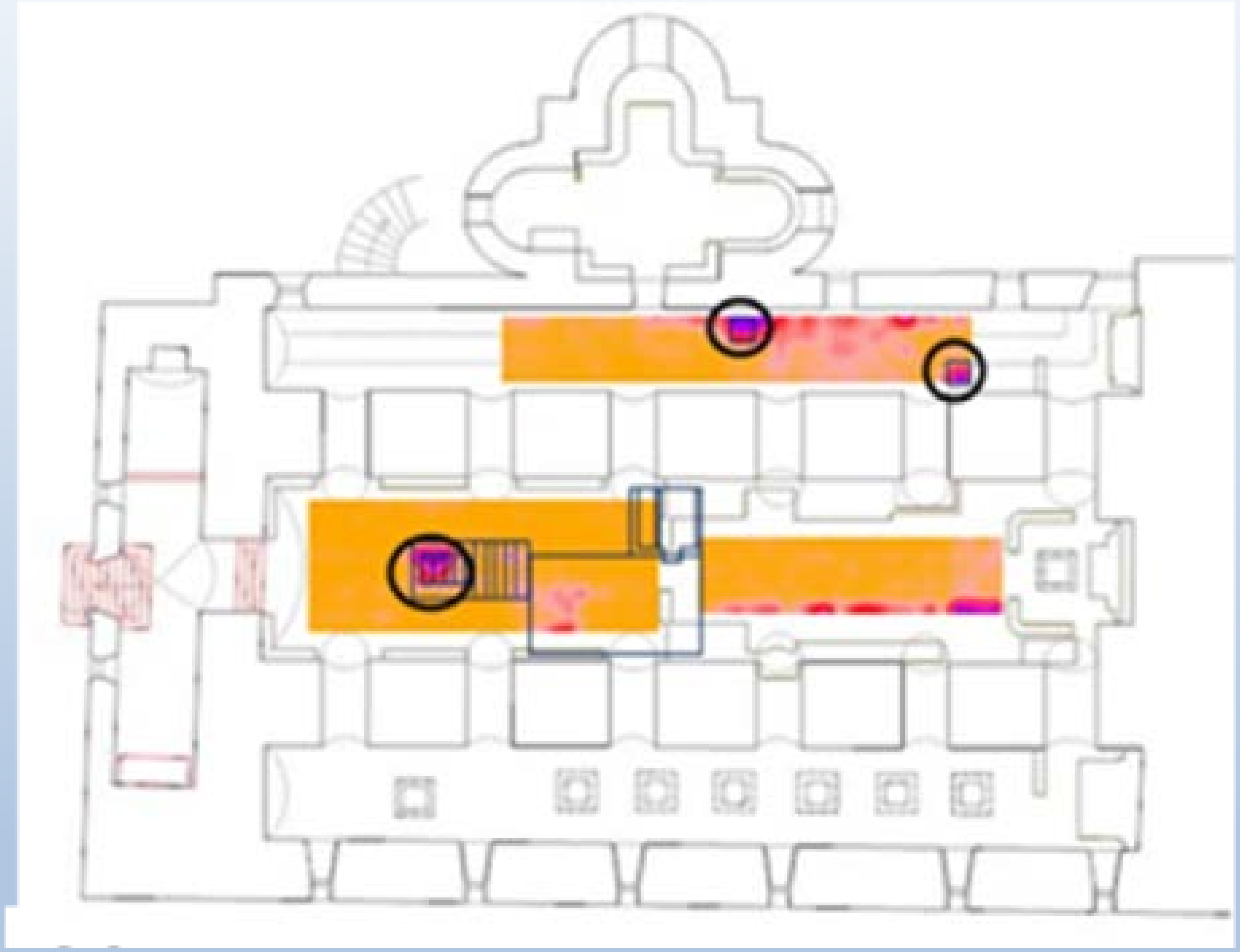
Depth 94 cm



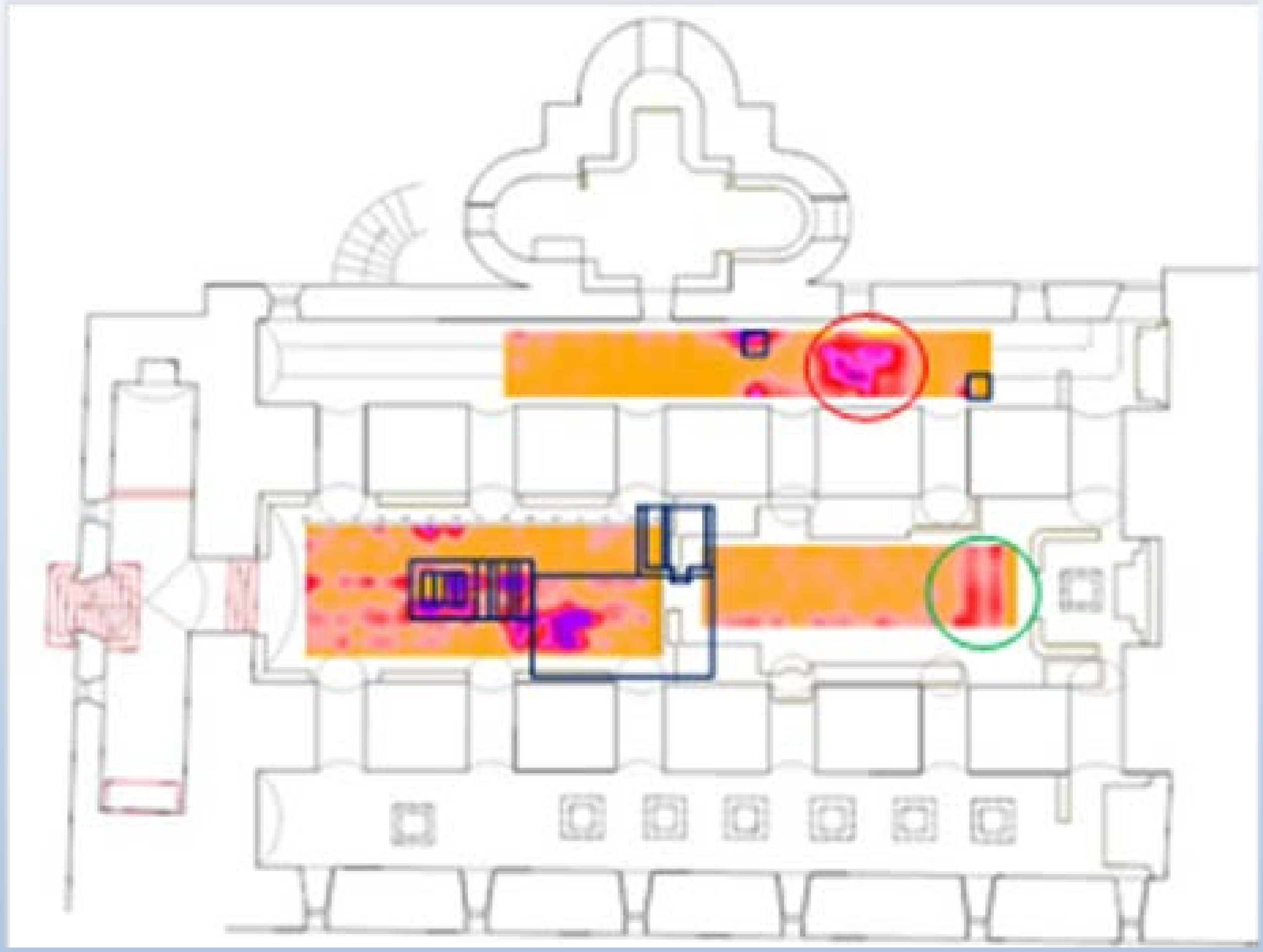
Depth 150 cm

The church of Santa Croce in Gravina in Puglia

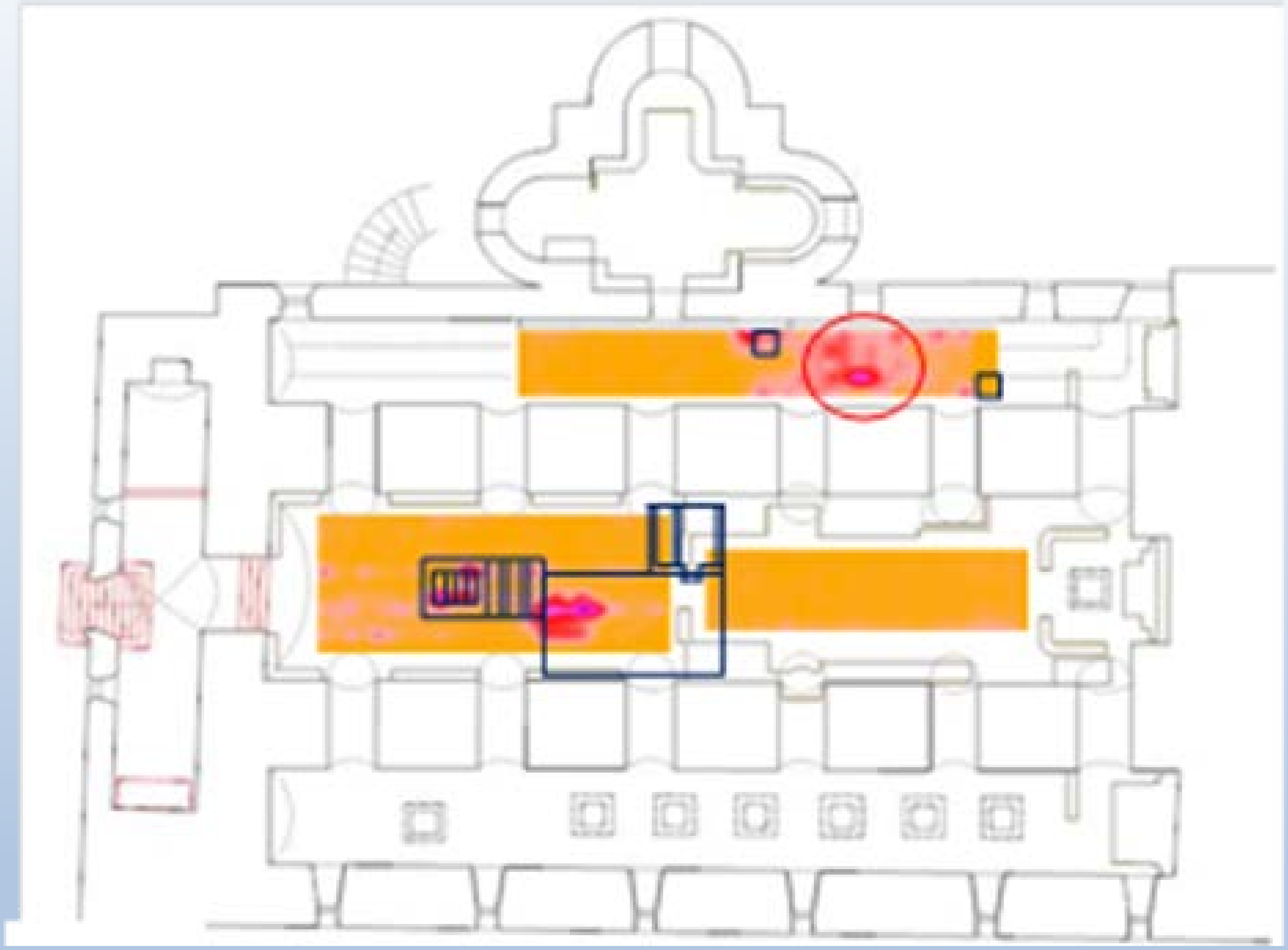




Depth 22 cm

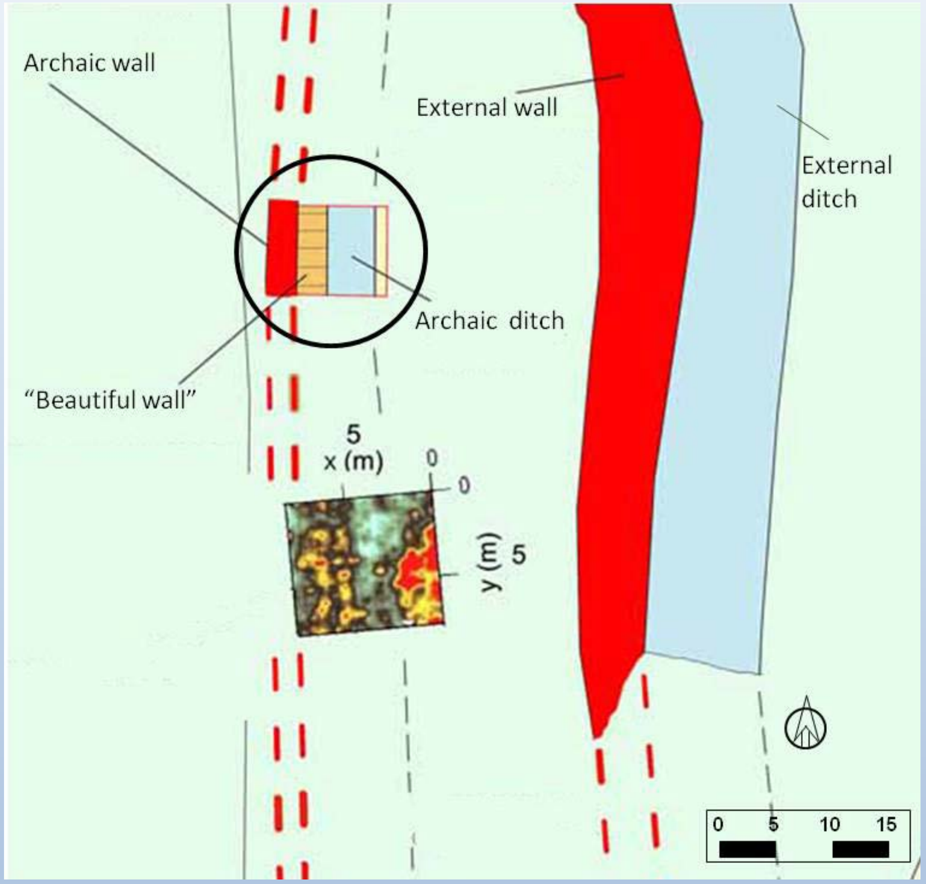


Depth 77 cm

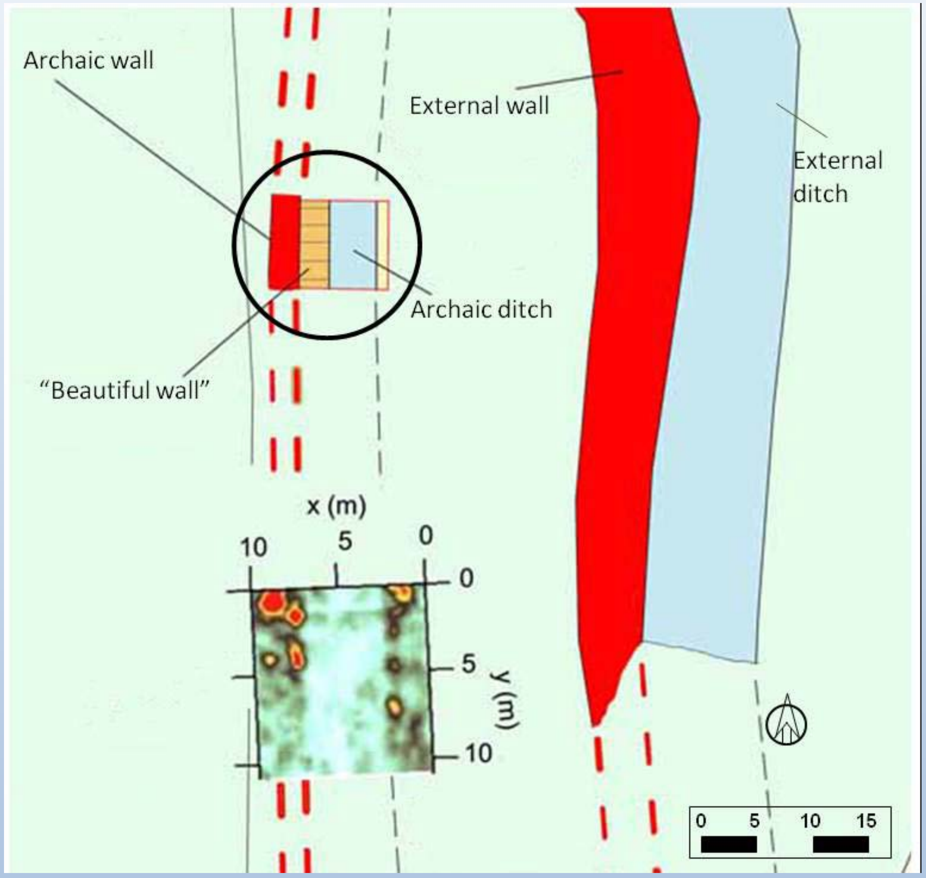


Depth 158 cm

The Archaic Ditch of Manduria



Pulsed GPR



Prototype

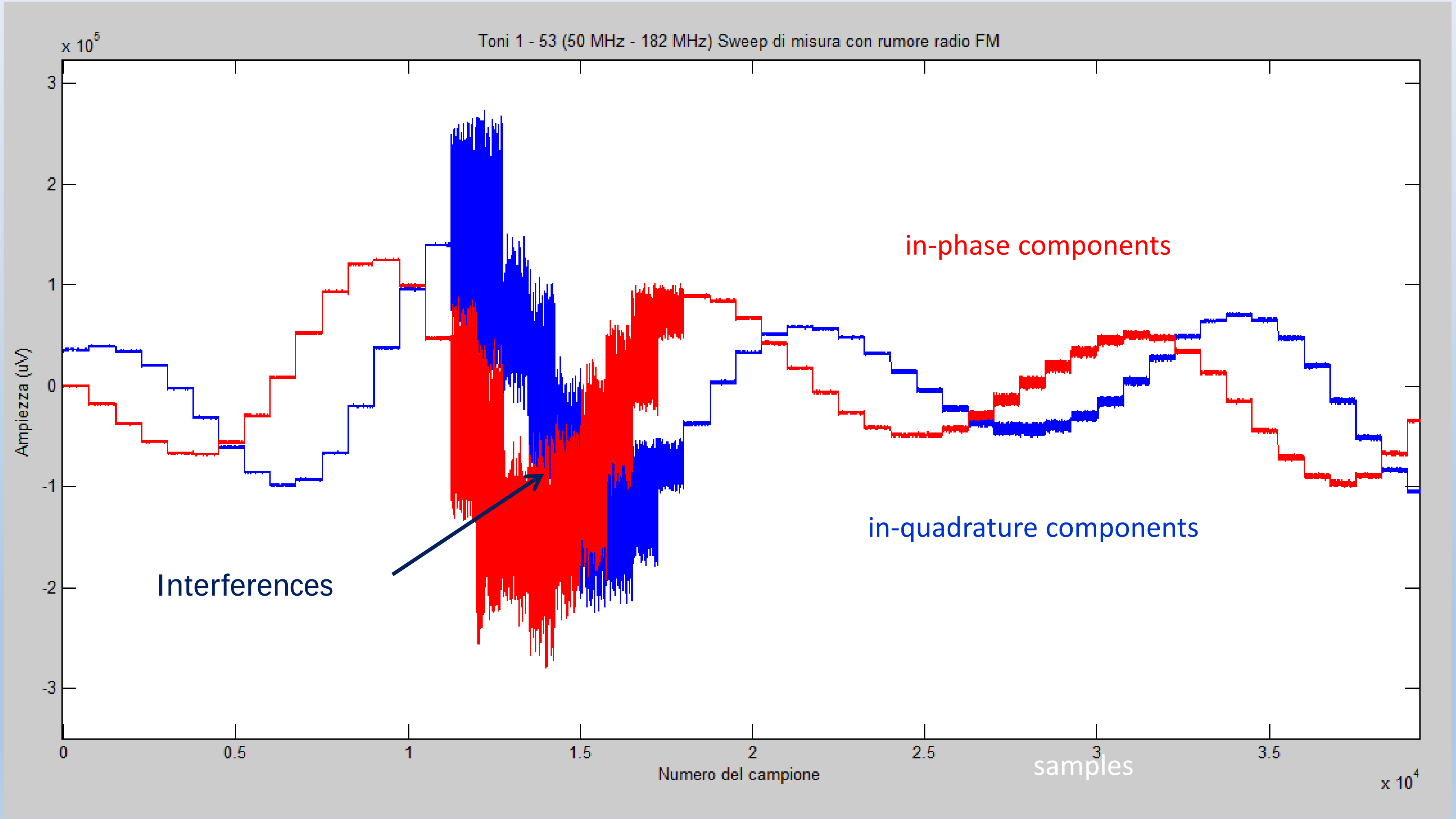
Depth 360 cm

Reconfigurable integration times

$$f_n \leftrightarrow T_{\mathrm{int}\,n}$$

Reconfigurable radiated power at each frequency

The reconfiguration of the integration times as strategy again interferences

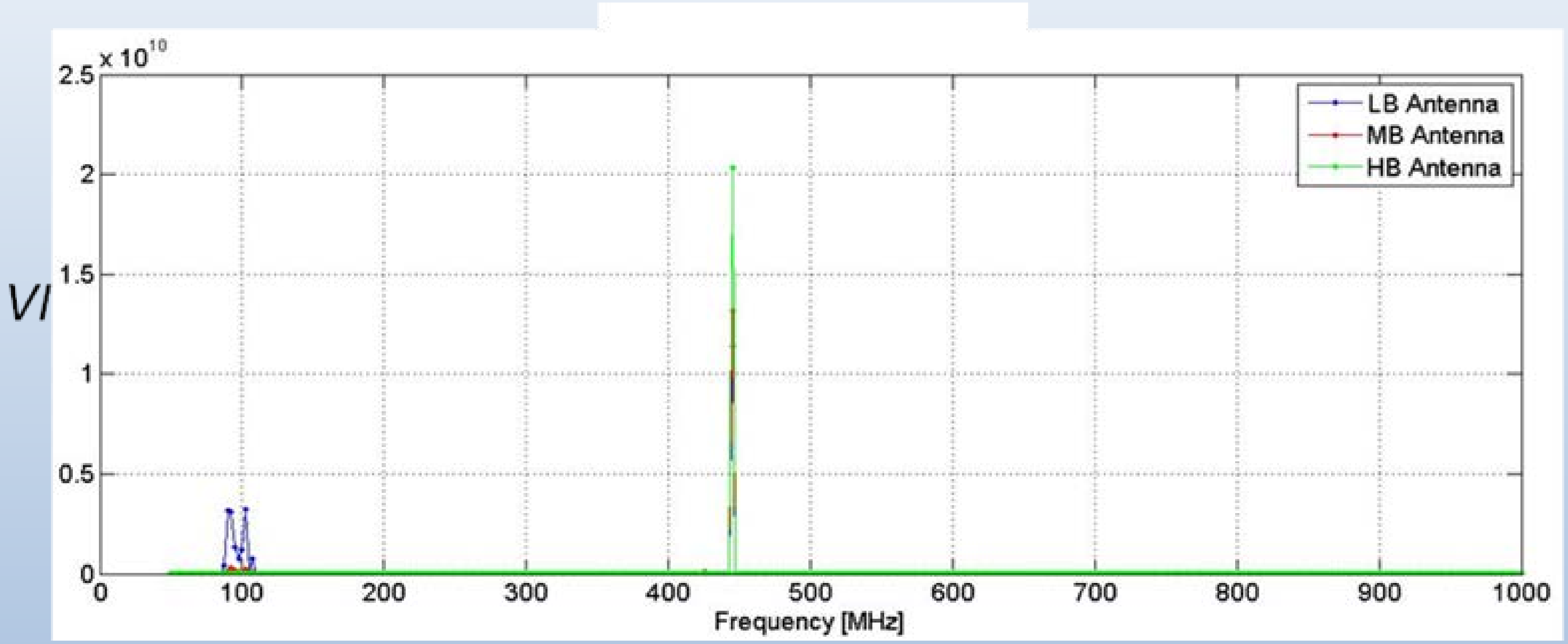


Variance of the samples

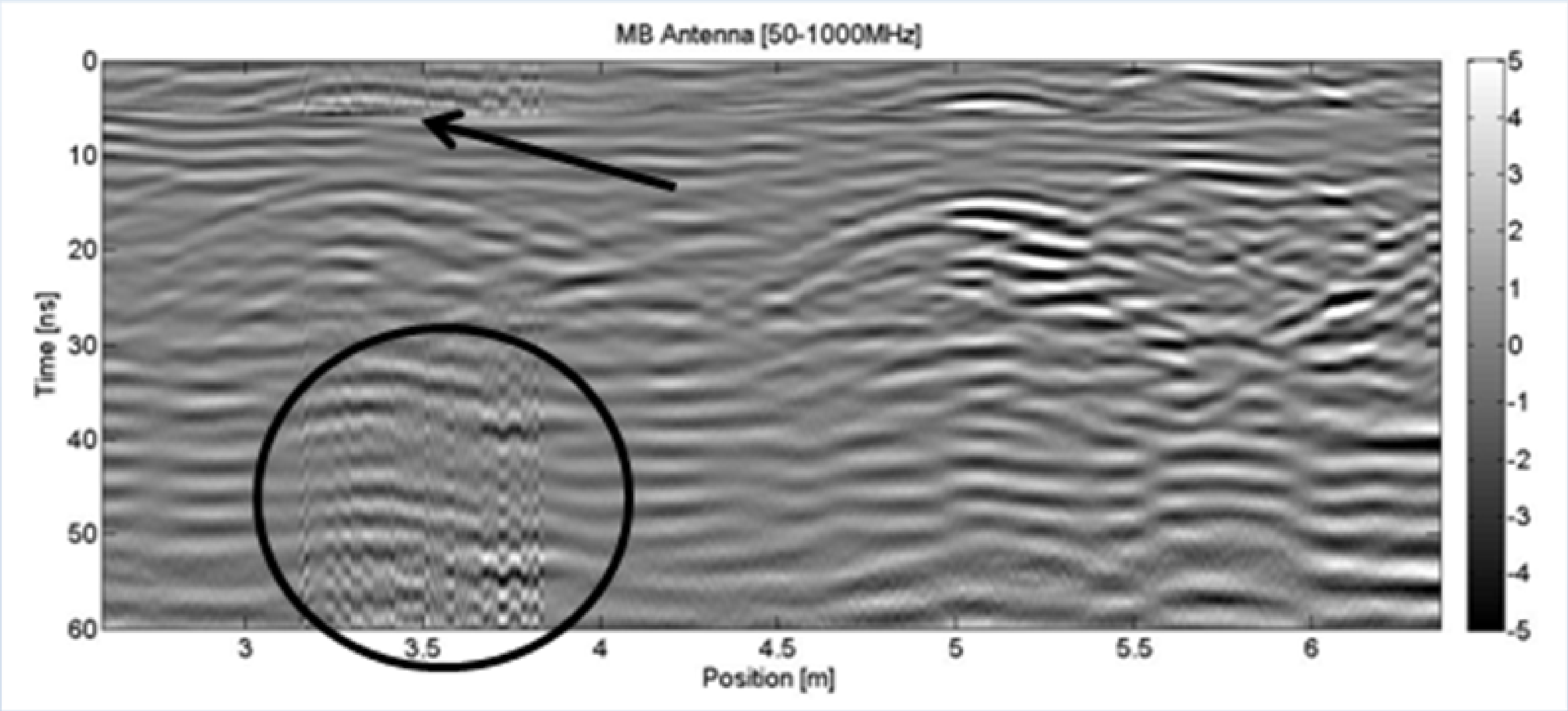
$$\begin{cases} \sigma_I^2 = \dfrac{I_1^2 + I_2^2 + \ldots I_N^2}{N} - \left(\dfrac{I_1 + I_2 + \ldots I_N}{N} \right)^2 \\ \sigma_Q^2 = \dfrac{Q_1^2 + Q_2^2 + \ldots Q_N^2}{N} - \left(\dfrac{Q_1 + Q_2 + \ldots Q_N}{N} \right)^2 \end{cases}$$

An experiment

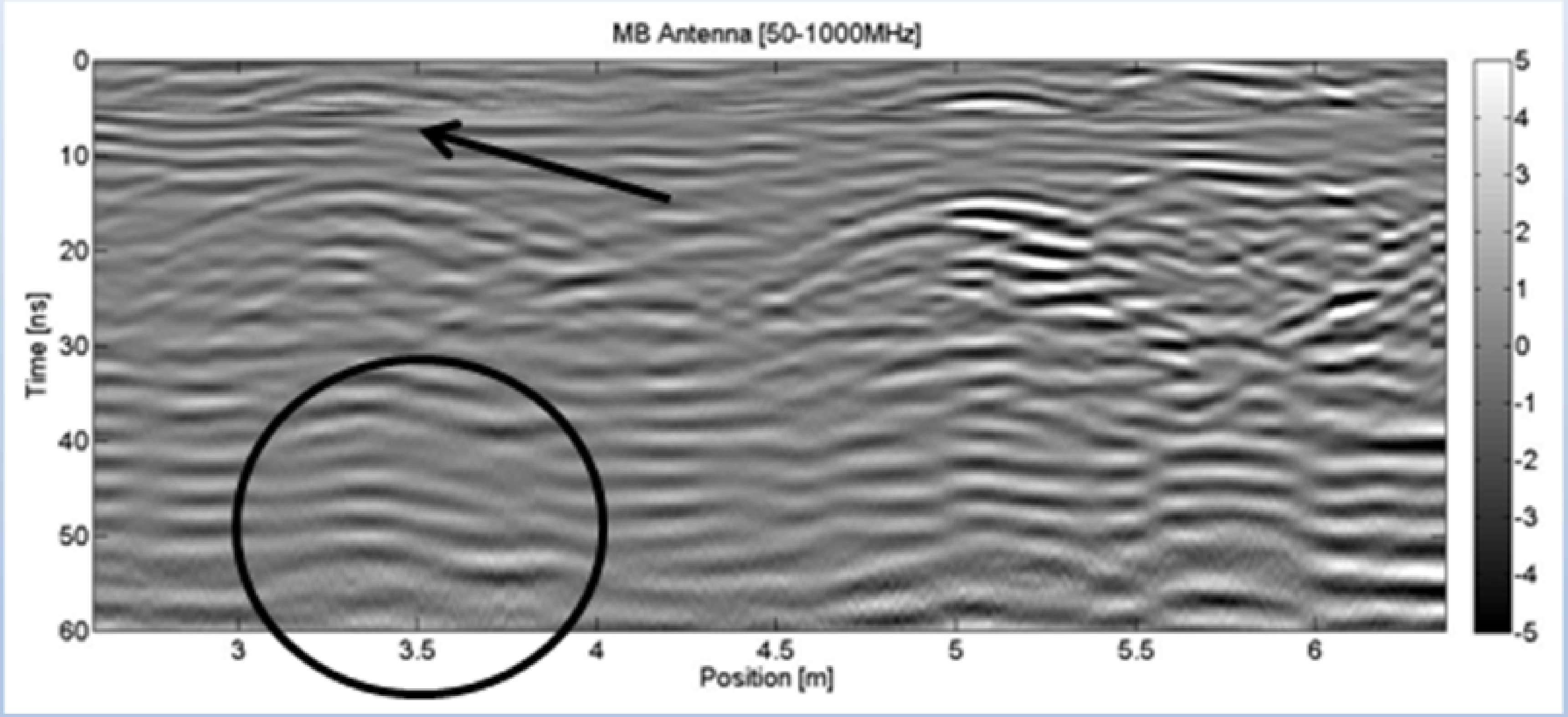
Interference of the transceivers



Signal with the default integration times

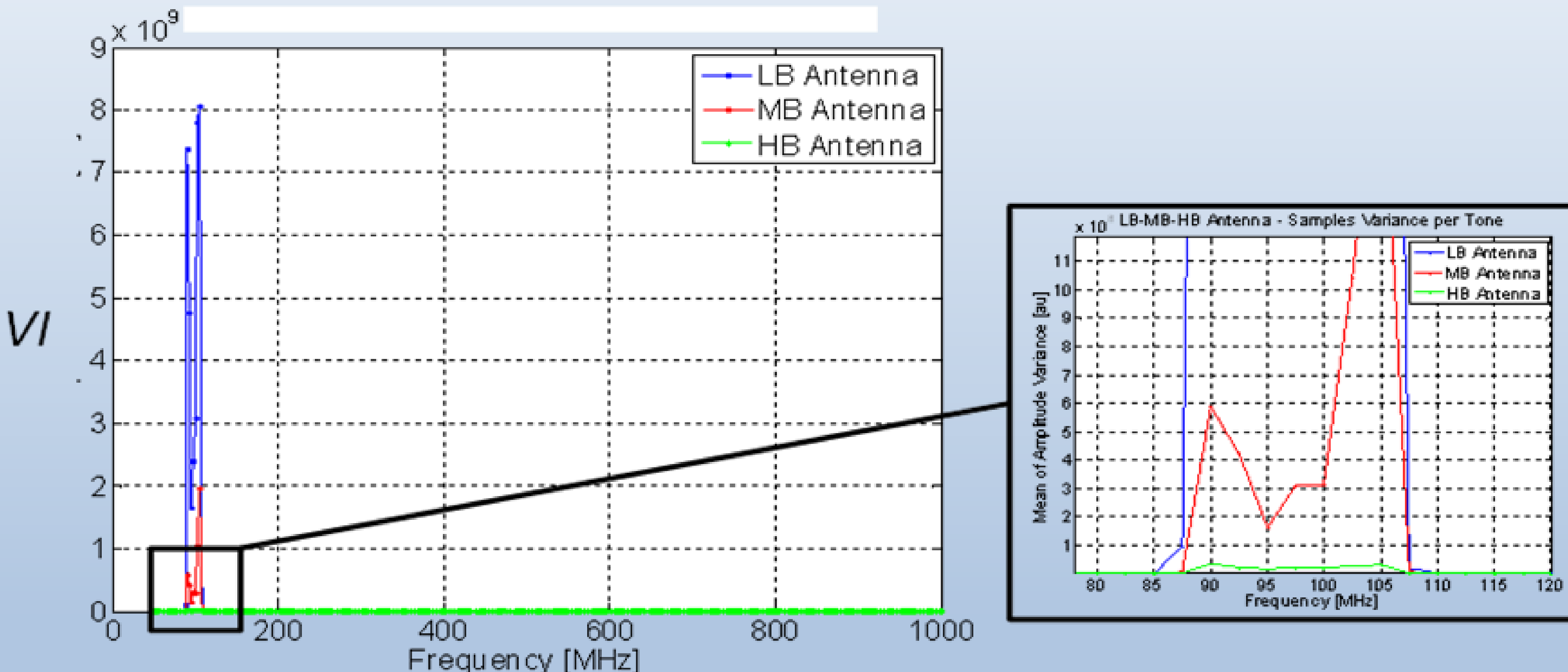


Signal with the reconfigured integration times

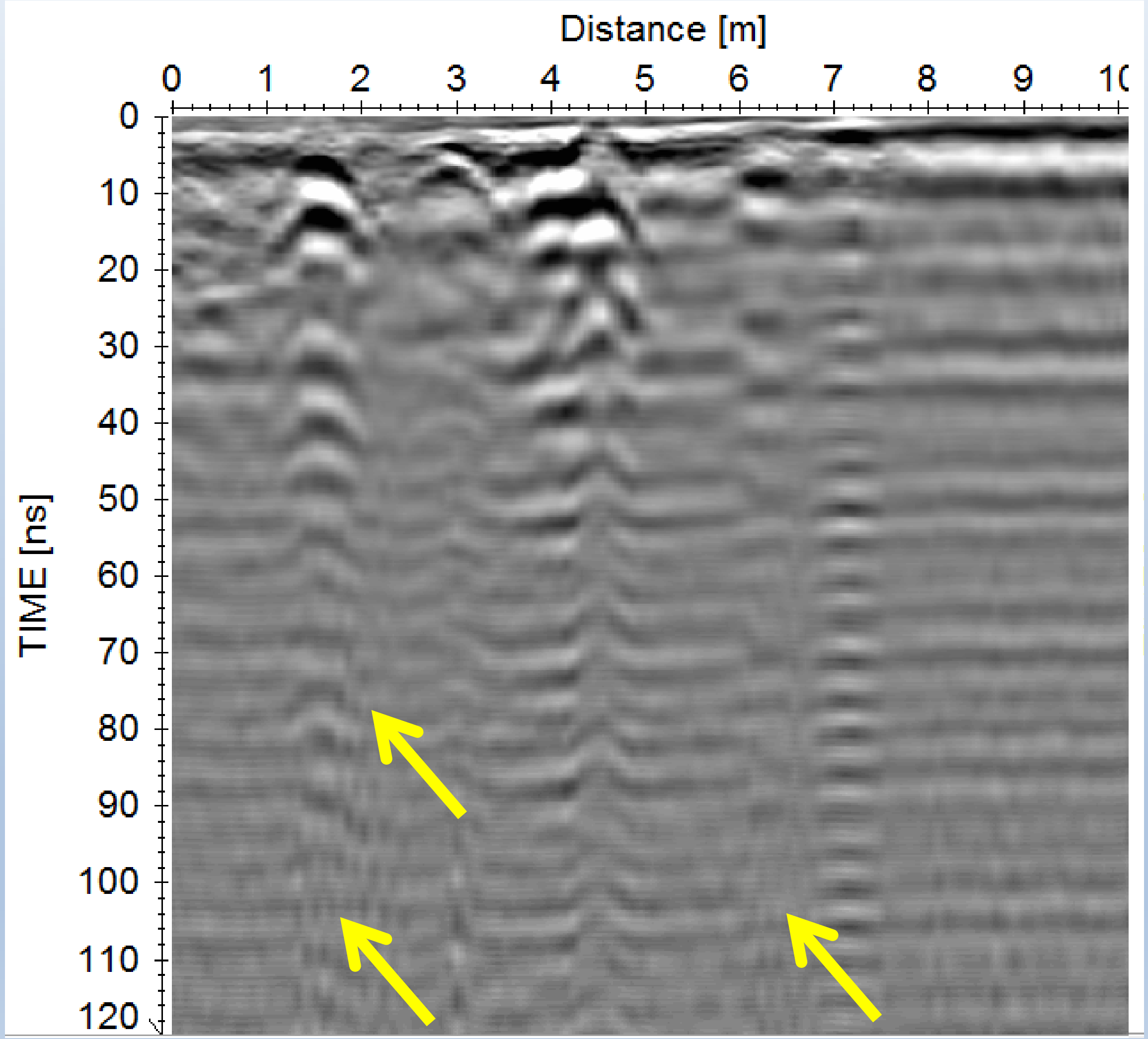


An experiment in the field

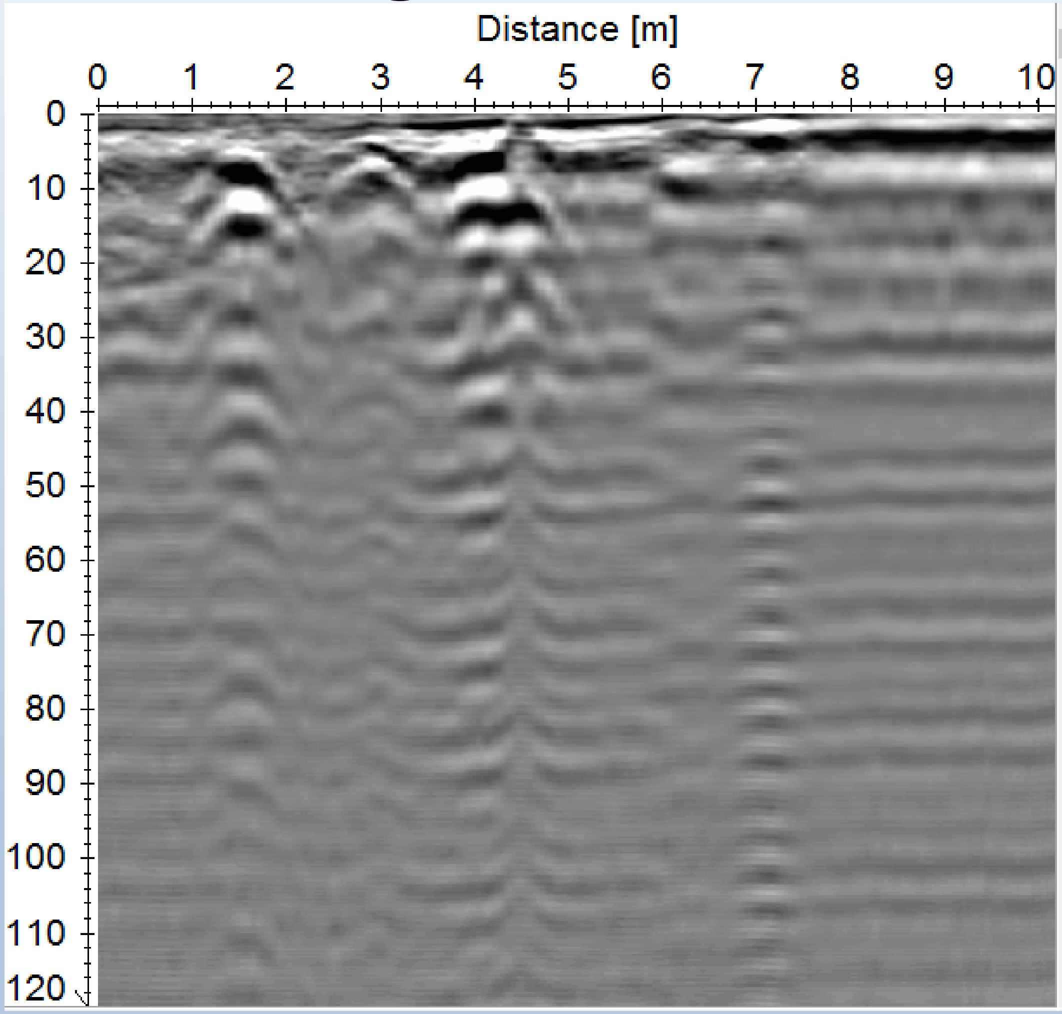
Interference of the repeater



Signal with the default integration times



Signal with the reconfigured integration times



Increasing of the comprehensive measurement time less than 5% in both cases.